DOCUMENT RESUME

ED 127 188 SE 021 225

AUTHOR	Ihrig, Elizabeth A., Ed.
TITLE	Using Larger Numbers: MINNEMAST Coordinated Mathematics - Science Series, Unit 20.
INSTITUTION	Minnesota Univ., Minneapolis. Minnesota School Mathematics and Science Center.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	71
NOTE	129p.; For related documents, see SE021201-234; Photographs may not reproduce well
AVAILABLE FROM	MINNEMAST, Minnemath Center, 720 Washington Ave., S.E., Minneapolis, MN 55414
EDRS PRICE	MF-$0.83 HC-$7.35 Plus Postage.
DESCRIPTORS	*Curriculum Guides; Elementary Education; *Elementary School Mathematics; *Elementary School Science; Experimental Curriculum; *Interdisciplinary Approach; Learning Activities; Mathematics Education; *Number Concepts; Primary Grades; Process Education; Science Education; Units of Study (Subject Fields)
IDENTIFIERS	*MINNEMAST; *Minnesota Mathematics and Science Teaching Project

ABSTRACT

This volume is the twentieth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by second-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of three groups of lessons and activities. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. This unit begins with three computational games, and then provides a sequence of thirteen lessons aimed at building skill at addition and subtraction with large numbers. The final set of lessons is related to building a weather station. (SD)

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SE 021 225

UNIT 20 — 2

MINNEMAST

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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USING LARGER NUMBERS

UNIT 20

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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The Minnesota Mathematics and Science Teaching Project developed these materials under a grant from the National Science Foundation.

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Second Printing
1971
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USING LARGER NUMBERS

This unit was developed by MINNEMAST on the basis of experiences of the many teachers who taught an earlier version in their classrooms.

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Complete List of Materials for this Unit

(Numbers based on class size of 30 and assumes the children play games in pairs.)

total number required to teach unit	item	lessons in which item is used
30	** Student Manuals	
15	* pair of dice, one marked 0,1,2,3,4,5 and one marked 0,-1,-2,-3,-4,-5	Game 1; 9
15	*** sets of Minnebars (optional)	Game 2; 5,9
45	sheets of paper	Game 2; 8,12
15	* $1\frac{1}{2}$ to 2" balls	Game 2
15	* spinners	Game 3; 9
15	* dials numbered from 0-9	Game 3; 9
600	** 1-inch squares	1
3-4	containers, e.g. shoe boxes	1
15	boxes or jars to shake squares (optional)	1
1	flannel board	2,7,10,11,12
30	shapes for flannel board or chalk board, 20 large, 10 small	2,7
30	*** student slide rules	3,5,6,9,11,12
1	demonstration number line (optional)	3
	counters (optional)	5,9
90	** sheets of $\frac{1}{2}$ inch grid on tagboard	6,7,8,10,11,12,13
30	envelopes (9" x 12")	6
1	* abacus	6,11,12,13
30	sheets of 12" x 20" construction paper or newsprint	7
1	roll of masking tape	7,8,10,11,12,13, 16,18
15	** decks of word problem cards	9
	numerals for flannel board (optional)	10,11,12

14	* thermometers	15,16
1	transparency of thermometer (see p. 117)	15
1	overhead projector	15
1	large, demonstration thermometer (optional) (see p. 77)	15
30	red crayons	15,16,17,19
1	* anemometer (see pp. 88-90)	17
1	watch or clock with second hand	17
1	transparency of wind-speed graph (see p. 119)	17
1	whistle (optional)	17
2	* plastic shoe boxes	18
200	* strips of blue cobalt chloride blotting paper, 1/4" x 2", and several longer strips	18
1	container of water and sponge	18
7-10	* soufflé cups containing: mineral oil, white corn syrup, vegetable oil, milk, pop, water, or other liquids	
1	felt-tip pen	18
1	* box of toothpicks	18
3	12-oz. plastic or glass containers	18
3	petri dishes, jar covers, or pieces of very heavy cardboard	18
1	* medicine dropper	18
30	** letters to Parents	18
1	hygrometer (see pp. 108-110)	19
1	transparency of humidity graph (see p. 121)	19

* kit items as well as

** printed materials available from Minnemath Center, 720 Washington Avenue S.E., Mpls., Minnesota 55455

*** available from The Judy Company, 310 North Second Street, Minneapolis, Minnesota 55401

INTRODUCTION

In past MINNEMAST units, the children have used a variety of techniques to solve addition and subtraction problems. These included using counters, numerals, number lines, the abacus and other devices. In Unit 20, we develop the standard algorithms (rules) for adding and subtracting multi-digit numbers, for example:

$$\begin{array}{r} 12 \\ -\ 1 \\ \hline 11 \end{array} \qquad\qquad \begin{array}{r} {\scriptstyle 1} \\ 16 \\ +27 \\ \hline 43 \end{array}$$

First the children use a place-value chart on which they manipulate counters to give concrete representations of the addition and subtraction methods. The counters are made so that a 10-piece is ten times as large as a 1-piece and a 100-piece is ten times as large as a 10-piece. The children trade ten 1-pieces for a 10-piece or a 10-piece for ten 1-pieces. After they have had sufficient practice in regrouping the counters, they learn to work with numerals instead of counters. They learn to record the process on paper in condensed form, rather than to use a place-value chart and counters or numerals. Thus they arrive at the standard algorithm.

This unit is divided into three sections. Section 1 is a group of games that provide practice in addition and subtraction. The games can be made more challenging as the children's skill increases. This flexibility means that the games can be used throughout the year without becoming repetitious.

Section 2 is devoted to teaching the standard algorithms for addition and subtraction. It begins with many activities designed to develop skill in using basic facts and then moves on to work with place value, since these two elements are basic to working with and understanding the algorithms.

In Section 3 the children learn to use different instruments to measure and study some of the components of weather. In the course of working with weather measurements, it is necessary for the children to do a great deal of computation, using the skills they acquire in Section 2. No attempt is made to teach about the weather. The children are only taught to use the measuring instruments.

NOTES ON TEACHING THIS UNIT

It is intended that Sections 2 and 3 be taught concurrently. The games in Section 1 should be presented early in the unit and should be returned to frequently as free time activities during and after the work in Sections 2 and 3. This unit should take approximately four weeks to complete.

SECTION I GAMES

Most children enjoy playing games. They can learn complex rules and develop strategies for winning without much difficulty. Games and informal activities can provide insight into and experiences with many mathematical concepts. Since the children enjoy the games, they will be more likely to develop good attitudes towards mathematics as well as to get substantial practice in computation.

Begin teaching this unit by introducing the simplest version of a game. You might introduce it during the mathematics period, but the children should be encouraged to play the games whenever they have extra time—when they finish some work ahead of the rest of the class, when they cannot go out for recess, or when they have time before school or at noontime. As the individual child's skill increases, guide him to more challenging versions of the games.

Game I: TUG-O-WAR

Version I of this game gives practice with addition and subtraction through back-and-forth moves on a number line according to the throw of dice.

Version II of the game involves addition and subtraction of a positive number and a negative number. This requires no preliminary review. The game can even serve as an independent teaching device. If a child rolls +2 and -4, he can simply move 2 steps in the positive direction and then 4 steps in the negative direction. After he has done this a number of times he should begin to use computation spontaneously as a short-cut.

MATERIALS

-- for each group of two --

- two dice, one marked 0, 1, 2, 3, 4, 5 (positive die) and one marked 0, -1, -2, -3, -4, -5 (negative die)
- marker
- game board (in Student Manual)

PROCEDURE

Version I

1. This game is played with a partner. The game board is at the beginning of the Student Manual; have the students tear it out carefully. Any small object such as a pencil or a narrow strip of colored paper can be used as a marker. Each team gets a pair of the negative-positive dice. First both players roll the positive die. The one who rolls the higher number will play with the positive die; the other will play with the negative die.

2. At the beginning of the game, the marker should be placed at 10 on the number line, the starting point. The player with the positive die rolls first. The number he rolls tells how many jumps he is to move the marker in the positive direction. Next the player with the negative die rolls.

He moves the same marker that many jumps back in the negative direction. As the children play, they should say aloud the addition or subtraction sentences that describe their moves. (For example, 10 plus 5 is 15; 15 minus 5 is 10; 10 plus 1 is 11; 11 minus 5 is 6, etc.)

3. The players alternately roll the dice and move the marker back and forth. The goal of the player who has the positive die is to get the marker up to 20. The goal of the player with the negative die is to get the marker down to 0.

4. The children may vary the rules of this game by setting a time limit or by limiting the total number of throws to 20. Whoever is closer to his goal at that time is the winner.

Version II

1. The players determine their goals by rolling the positive die. The one who throws the higher number has 20 as his goal and goes first; the player with the lower number works toward 0.

2. Each player rolls both dice. He moves, according to the throw, the given number of moves in each direction. His total move may place the marker farther from his own goal.

3. The winner is the player who reaches his own goal first. If a player reaches his opponent's goal, the opponent wins.

4. A time limit or a set number of moves may be used to end the game, in which case the player nearer his goal is the winner.

Game 2: MINNEBOWLING

This game is played by 2, 3 or 4 players who take turns rolling a ball and knocking over Minnebars. Each player adds together the number of Minnebar units he knocked over. The first player to reach 50 is the winner. Scoring may be done any of a number of ways.

MATERIALS

-- for each group --

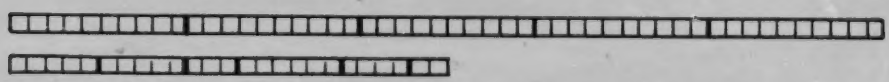
- Minnebars
- blank sheet of paper
- 1½ inch ball

PROCEDURE

The players lay a blank piece of paper on the floor, and arrange on it any number of Minnebars standing on end. Then they move back a distance of about 3 feet. (They may want to mark this distance with a chalk line.) They take turns rolling a small ball in an attempt to knock down the Minnebars. Each player removes the bars he knocks over and records his score. His score is the sum of the number of units of the Minnebars he knocks over. For example, if he knocks over a 5-unit Minnebar, a 2-unit Minnebar and an 8-unit Minnebar, his score for that try is 15. One round is over when all the bars are knocked down. Then the players set up the bars again and the game continues until one player has reached the goal of 50.

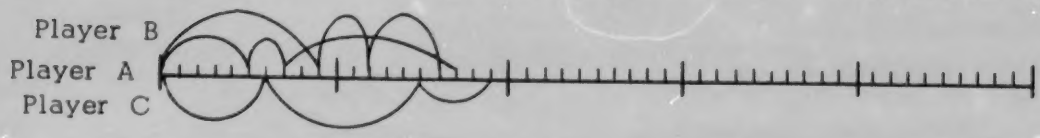
Scoring Methods

1. Use another set of Minnebars and lay out five 10-unit Minnebars end to end to represent 50. As each player knocks over Minnebars, he lays them end to end until his bars total 50 or more. (He compares the length of his string of Minnebars to the original 50-unit string of Minnebars.)



Since this method of scoring removes the need for adding up the score, only the slowest students should use it.

2. A number line from 1 to 50 is drawn on the chalkboard. As a player knocks down Minnebars, he records his score on the number line, adding the value of each bar to his previous sum. Each child should use a different color of chalk.



3. Each player keeps score on a piece of paper, adding each new number to his previous total.

The players may choose some number other than 50 as their goal.

Game 3: SPINNER WINNER

This game is played by 2, 3 or 4 players who take turns spinning a spinner to generate numbers. Variations in scoring procedure give practice in addition and subtraction.

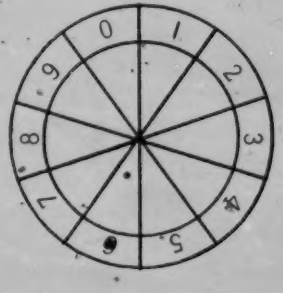
MATERIALS

-- for each group --

- spinner
- 1 dial numbered from 0 to 9

PREPARATION

Before you introduce this game, mark the spinner dials. For Versions I through IV, the dial should be marked like this:



PROCEDURE

Version I

The players choose some number to be the goal for this game. Then they take turns spinning the spinner and each player records the numbers he spins. As the players continue, each one keeps a running total of his own score. They might keep score on number lines, or each new number might be added to the previous total in column form.

The winner is the player whose final score comes closest to the goal without exceeding it. If a player has a score which is very close to the goal (one or two points below it) he may decide not to spin. However, if he does so, he forfeits his right to spin again during that game. When each player has either exceeded the goal number, reached the goal number, or decided not to spin, the game is over.

Version II

The players choose a goal number (for example, 40) and take turns spinning the spinner. They do not keep individual scores; rather, each player adds his number to the sum of all previous numbers spun by the group. For example, if Player A spins 5, and Player B spins 7, B records 12 as the running total. If Player C spins 6, he adds it to 12 for a running total of 18. The winner of the game is the last person to spin before the running total becomes larger than the goal number. The players keep score on a number line or they could add up the running total in vertical form.

Version III

Rather than starting at 0 for Versions I and II and working towards a goal number, the players could start from some number (for example, 40) and work backwards toward 0. This way, the children will be able to practice subtraction.

Version IV

The players choose a goal number and alternately spin the spinner and write down their numbers on a slip of paper. After all the players have had five turns, each one adds together his five numbers. The player whose total is nearest to the goal number (either above or below it, if not on it) is the winner. A more advanced variation would have a larger number for a goal, with each player getting a larger number of turns.

Version V

After the class has had Lesson 11, substitute a spinner dial numbered from 10 to 19 for greater challenge. You might want to make a dial with random two-digit numbers, for example, 11, 22, 49, 13, etc.

SECTION 2 COMPUTATIONAL SKILLS

PURPOSE

- To review basic addition and subtraction facts.
- To review and extend previous work with place value.
- To introduce the algorithms for addition and subtraction.
- To introduce regrouping in addition and subtraction.

Lesson 1: REINFORCING BASIC ADDITION AND SUBTRACTION FACTS

This lesson provides review and reinforcement of the basic addition and subtraction facts. It also reviews the commutative and associative properties of addition. The students record their work in several ways. Then a game provides substantial addition and subtraction practice.

MATERIALS

- 1-inch squares, 20 per child
- 3 or 4 empty containers (e.g., shoe boxes)
- Worksheet 1

PROCEDURE

Activity A

Write an incomplete addition sentence such as $3 + 2 = \square$ on the chalkboard. Ask a child to come to the board and write the same information in another form. He might write $\square = 3 + 2$. Have different children try to think of other ways to write this information. (Many will be other than the standard forms.) Some possible forms are:

$3 + 2 = \square$ $\quad \begin{array}{r} 3 \\ +2 \\ \hline \square \end{array}$ $\quad \begin{array}{r} 2 \\ +3 \\ \hline \square \end{array}$ $\quad \begin{array}{r} \square \\ \hline 2 \\ +3 \end{array}$ $\quad \begin{array}{r} \square \\ \hline 3 \\ +2 \end{array}$

$2 + 3 = \square$ $\quad \square = 3 + 2$ $\quad \square = 2 + 3$

Next to the various addition sentences that were written on the chalkboard, write a subtraction sentence such as $7 - 3 = \square$. Again have children write the sentences in

different forms. Some possible forms are:

$7 - 3 = \square$ $\qquad$ $\begin{array}{r} 7 \\ -\ 3 \\ \hline \square \end{array}$ $\qquad$ $\begin{array}{r} 3 \\ +\ \square \\ \hline 7 \end{array}$ $\qquad$ $\begin{array}{r} \square \\ +\ 3 \\ \hline 7 \end{array}$

$\square = 7 - 3$ $\qquad$ $3 + \square = 7$ $\qquad$ $\square + 3 = 7$

$7 = \square + 3$ $\qquad$ $7 = 3 + \square$

Activity B

Give each child twenty 1-inch squares. The children should turn their squares so that the white sides are up, write an addition or subtraction combination on the white side of each square, turn the square over and write the answer on the yellow side. Some examples are:

white side	yellow side	white side	yellow side
3+4	7	11−7	4

Encourage the children to write both easy and difficult combinations on their squares. You might ask them to use all the digits from 0 to 9 in forming their combinations.

If a student finishes all his squares before the rest of the class, he could spend some time looking at the white side and predicting what is on the yellow side.

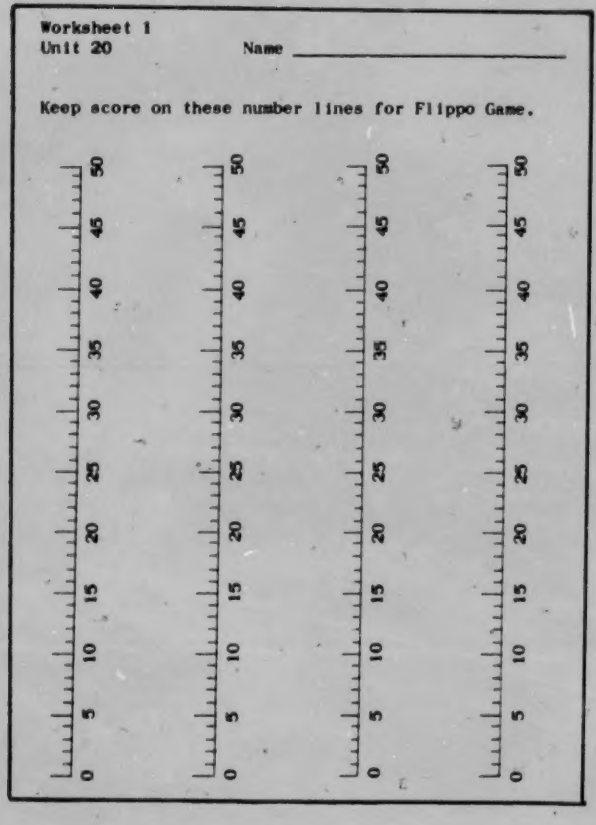
When most of the children have finished, have them place all their squares in a big box or in a few smaller boxes such as shoe boxes. Now they are ready to play the game "Flippo."

Flippo

This game can be played by 2 or 3 students. They begin by reaching into a box containing 1" squares and take out about 20 squares. Player A mixes the squares by shaking them in a plastic cup or in his hands. Then he dumps them onto a desk top. Player B finds all the squares that are white side up and tries to give the answer for each of them. If he gives the correct answer, he places the square in his win pile. If he gives the incorrect answer, he puts that square with those that fell yellow side up.

After he is finished with all the white squares, Player B puts those he missed back in the shaker, together with the squares that had fallen yellow side up. He scatters them on the desk top and then Student A (or C) has his chance to give answers for the squares that fall white side up.

After all the squares have been answered correctly, each player counts the number of squares in his win pile and marks his score on a number line on Worksheet 1.


Worksheet 1
Unit 20 Name _____

Keep score on these number lines for Flippo Game.

0 5 10 15 20 25 30 35 40 45 50

0 5 10 15 20 25 30 35 40 45 50

0 5 10 15 20 25 30 35 40 45 50

0 5 10 15 20 25 30 35 40 45 50

Now it is Player B's turn to pick up all the pieces and shake them. Player A gets first try at answering. When they finish this round, they add their second-round scores to their first-round score on Worksheet 1.

Different rules for deciding the winner may be used:

1. The one with the most points after 6 rounds is the winner.
2. The student who reaches 50 first is the winner.
3. Begin scoring at 50 and go backward to 0. (The children will have to subtract their scores instead of adding them.) The first to reach 0 wins.

This game should be played several times. For each game, the students should exchange their squares for a new assortment. In this way, they will eventually work with most of the squares.

Activity C

The same squares may also be used in other ways. In their spare time, students may try these games:

1. A pair of children line up several chains of 20 squares, white side up. One child begins supplying answers. If he gets through a chain with no mistakes, he scores 20 points, and starts on the next chain. If he misses, he gets no points, and the other child gets to try for 20. The child with the highest score wins. (If this is too frustrating for some children, have them work with chains of 10 squares.)

2. A child collects his own set of squares, in which he includes those he finds most difficult to answer. He lines them up, white side up, and practices with them. When he feels ready, he challenges another child to a contest using those squares.

Lesson 2: MATHEMATICAL PATTERNS

The children are given experience with several mathematical patterns. Many of the worksheets are dot pictures, for which the children first solve equations to find the names of the dots, and then connect the dots according to the appropriate pattern. This work provides practice in addition and subtraction and experience in recognizing patterns.

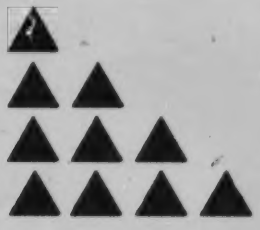
MATERIALS

- flannel board and about 20 shapes (if flannel board is not available, use chalkboard)
- Worksheets 2 through 7

PROCEDURE

Activity A

On a flannel board, set up the following pattern of objects.



HOW MANY OBJECTS SHOULD I PUT IN THE NEXT ROW IN ORDER TO CONTINUE THIS SAME PATTERN? (5.)

Ask the children to explain the pattern. (Each succeeding row has 1 more object.)

Set up the following patterns of objects and ask similar questions.

0, 000, 00000, ...
(Each successive set has 2 more objects.)

000000, 00000, 0000, ...
(Each successive set has 1 less object.)

0000000, 00000, 000, ...
(Each successive set has 2 less objects.)

You might give more difficult patterns if the class seems to do these easily. For example:

0, 000, 00, 0000, 000, ...
(The pattern is: increase by 2, decrease by 1, increase by 2, etc.)

Activity B

Place the following sequences of numbers on the chalkboard one at a time. Ask the children to complete each sequence as you write it on the board. Have them explain the patterns.

1, 2, 3, 4, __, __, __, __

1, 1, 2, 2, 3, 3, __, __, __, __, __, __

0, 2, 4, __, __, __, __

10, 9, 8, __, __, __, __, __

0, 3, 6, __, __

16, 13, 10, 7, __, __

Numbers need not be used exclusively in this activity.

A, AB, ABC, ___, ___

X, XXX, XXXXX, _____

A, 1, B, 2, __, __, __, __

Encourage the children to think of patterns of their own to put on the board for other children to complete.

Worksheet 2
Unit 20 Name _____

Complete the patterns.

0, 1, 2, 3, 2, 1, 0, 1, 2, 3, 2

AA, B, CC, D, EE, F, GG

1, 3, 5, 7, 9, 11

10, 8, 6, 4, 2, 0

1, 4, 7, 10, 13, 16

11, 10, 9, 8, 7, 6, 5

1, 3, 2, 4, 3, 5, 4, 6, 5, 7

1, 2, 4, 7, 11, 16, 22

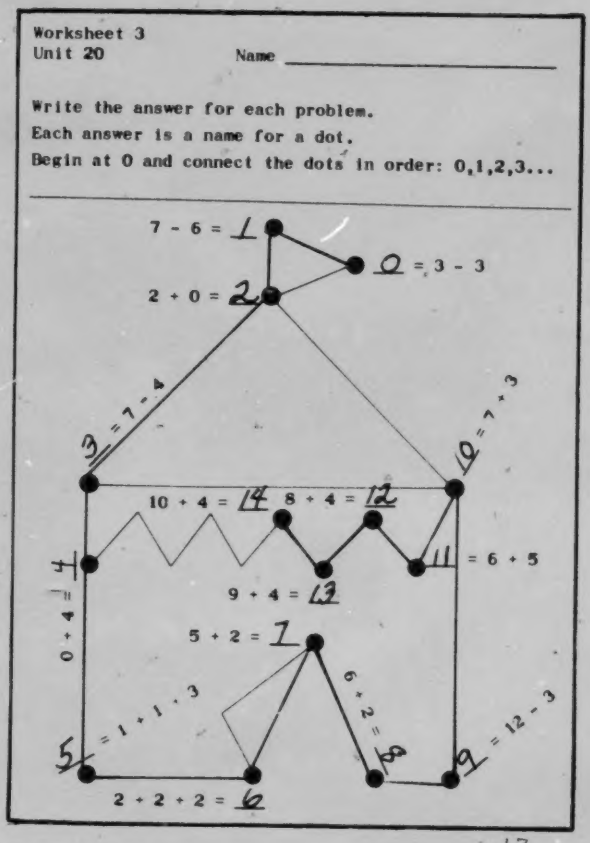
Make up your own pattern.

—, —, —, —, —, —

Have the children turn to Worksheet 2. This is an extension of Activities A and B. The patterns vary in difficulty. At the bottom of the worksheet, the children can make up a pattern of their own.

Activity C

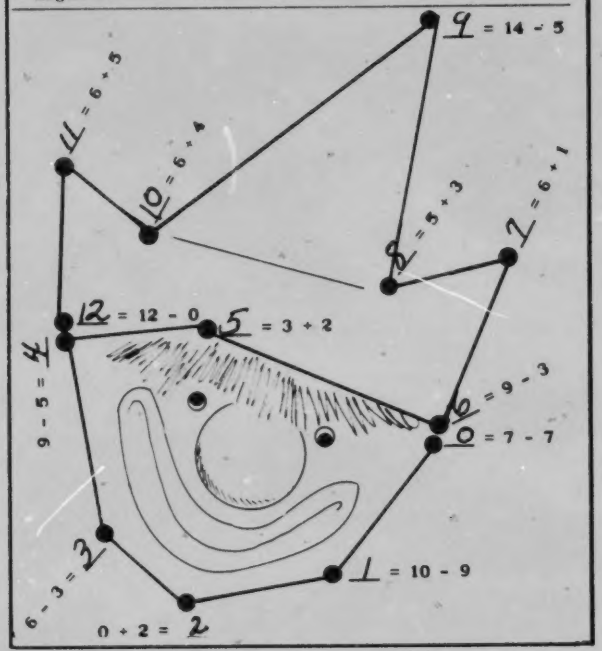
On Worksheets 3, 4, 5, 6 and 7, the students are to complete various mathematical sentences. Using the answers as the names for the dots, the students connect the dots. Worksheets 3, 4 and 5 start at 0 and go on in counting order. Worksheet 6 starts at 2 and counts by 2's. Worksheet 7 starts at 1 and goes by 1's. Worksheets 3 and 4 should be done during class time. Worksheets 5, 6 and 7 are provided for extra practice and can be done during free time.


Worksheet 3
Unit 20 Name
Write the answer for each problem.
Each answer is a name for a dot.
Begin at 0 and connect the dots in order: 0,1,2,3...
7 - 6 = 1
0 = 3 - 3
2 + 0 = 2
3 = 7 - 4
10 = 7 + 3
10 + 4 = 14
8 + 4 = 12
11 = 6 + 5
9 + 4 = 13
5 + 2 = 7
0 + 4 = 4
5 = 1 + 1 + 3
6 + 2 = 8
9 = 12 - 3
2 + 2 + 2 = 6

17

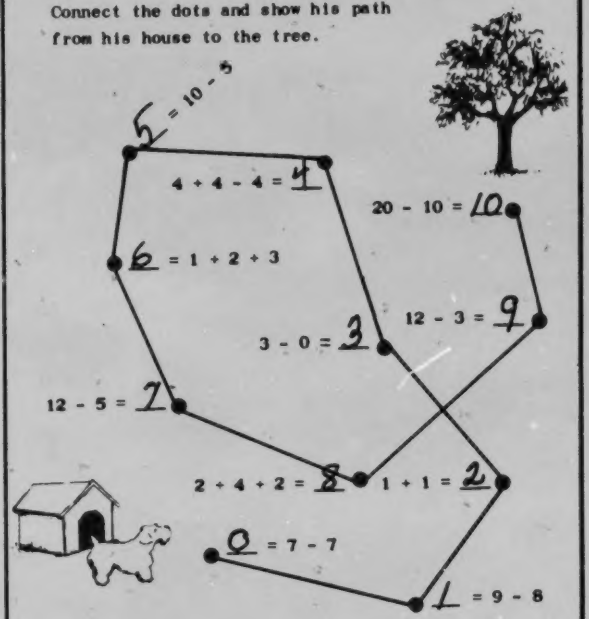
Worksheet 4
Unit 20 Name _____

Write the answer for each problem.
Each answer is a name for a dot.
Begin at 0 and connect the dots in order: 0,1,2,3...



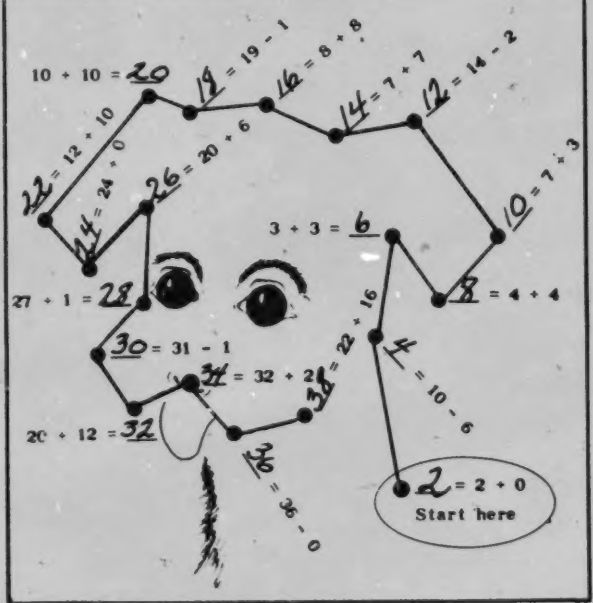
Worksheet 5
Unit 20 Name _____

The dog ran from his house to 7 - 7.
He ran to 9 - 8. Then he ran to 1 + 1.
Connect the dots and show his path
from his house to the tree.



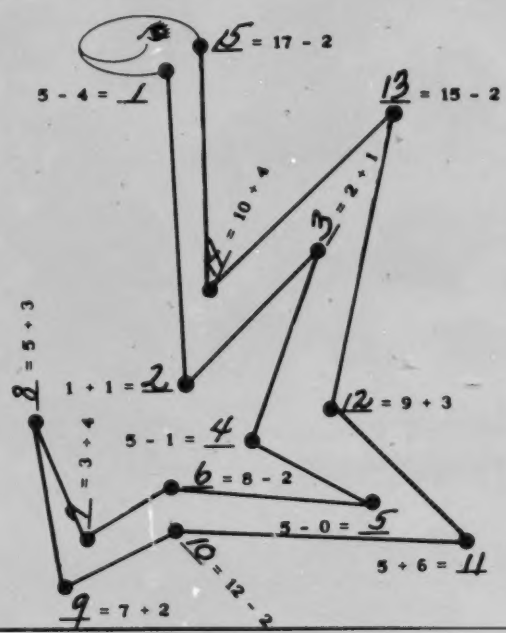
Worksheet 6
Unit 20 Name _____

Work the problems.
Can you find the number pattern?
Connect the dots.



Worksheet 7
Unit 20 Name _____

Each answer is a name for a dot.
Begin at 1 and connect the dots.



Lesson 3: FINDING THE ADDITION AND SUBTRACTION RULE

In this lesson the children determine the operation used in going from one number to another. The lesson provides experience with the concept of function and more practice in addition and subtraction.

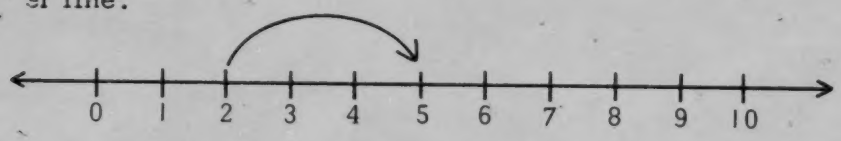
MATERIALS

- student number lines
- demonstration number line (optional)
- Worksheets 8 through 12

PROCEDURE

Activity A

Each child should have a number line on his desk. Use a demonstration number line or draw a number line (0 to 10) on the chalkboard. Draw an arrow from 2 to 5 on the number line.



HOW CAN WE GO FROM 2 TO 5? (Move right 3 spaces.)

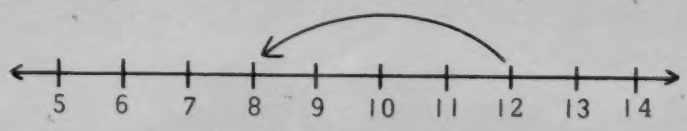
WHAT OPERATION MIGHT THIS MOVE REPRESENT? (Addition.)

Write 4 ⟶ 7 on the chalkboard.

WHAT RULE COULD WE USE FOR BOTH PROBLEMS, 2 ⟶ 5 and 4 ⟶ 7? (Add 3 to the first number to get the second number.)

IF WE USE THIS SAME RULE, HOW WOULD WE COMPLETE THE FOLLOWING? 5 ⟶ ☐, 6 ⟶ ☐, 3 ⟶ ☐

Now mark the number line as shown below.



HOW DO WE GO FROM 12 TO 8? (Move 4 spaces to the left.)

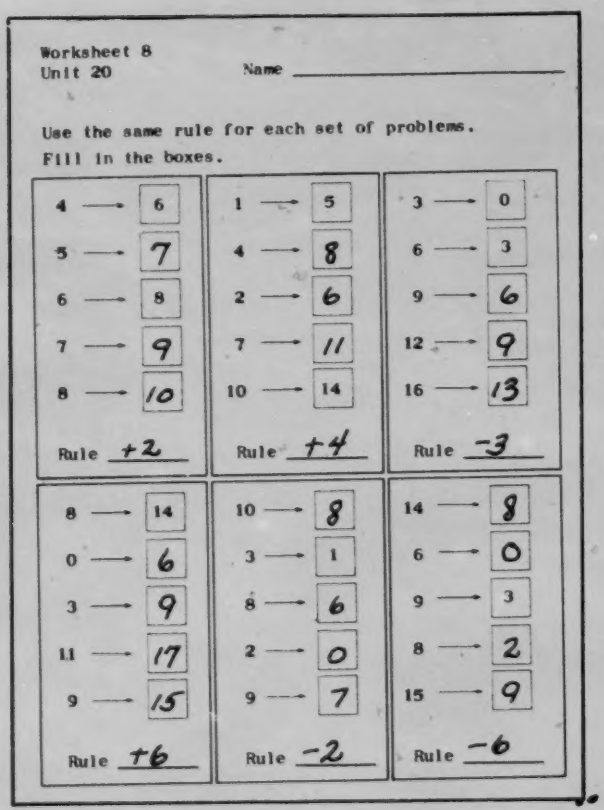
WHAT OPERATION MIGHT THIS MOVE REPRESENT? (Subtraction.)

Write 12 → 8 and 10 → 6 on the chalkboard.

WHAT RULE IS USED TO GET FROM THE FIRST NUMBER TO THE SECOND IN BOTH PROBLEMS? (Subtract 4 from the first number to get the second.)

Ask the children to use the rule to complete problems such as 13 → ☐, 9 → ☐, 4 → ☐. Use as many examples as you feel are necessary. Allow the children to use their desk number lines to help find the solutions.

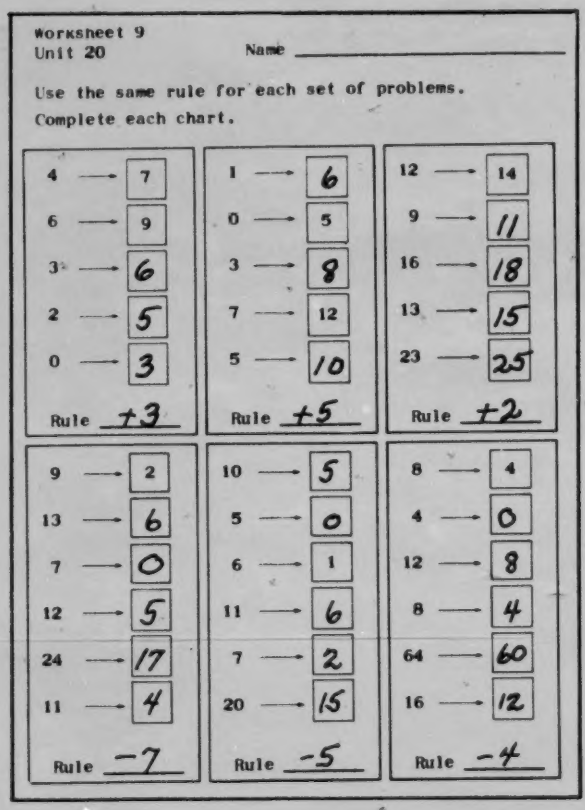
Have the children complete Worksheet 8. They should indicate what the rule is on the line below each group of problems.



Worksheet 8
Unit 20 Name _____

Use the same rule for each set of problems.
Fill in the boxes.

4 → 6	1 → 5	3 → 0
5 → 7	4 → 8	6 → 3
6 → 8	2 → 6	9 → 6
7 → 9	7 → 11	12 → 9
8 → 10	10 → 14	16 → 13
Rule +2	Rule +4	Rule -3
8 → 14	10 → 8	14 → 8
0 → 6	3 → 1	6 → 0
3 → 9	8 → 6	9 → 3
11 → 17	2 → 0	8 → 2
9 → 15	9 → 7	15 → 9
Rule +6	Rule -2	Rule -6



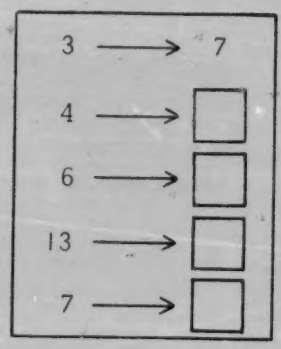
Worksheet 9
Unit 20 Name _____

Use the same rule for each set of problems.
Complete each chart.

4 → 7	1 → 6	12 → 14
6 → 9	0 → 5	9 → 11
3 → 6	3 → 8	16 → 18
2 → 5	7 → 12	13 → 15
0 → 3	5 → 10	23 → 25
Rule +3	Rule +5	Rule +2
9 → 2	10 → 5	8 → 4
13 → 6	5 → 0	4 → 0
7 → 0	6 → 1	12 → 8
12 → 5	11 → 6	8 → 4
24 → 17	7 → 2	64 → 60
11 → 4	20 → 15	16 → 12
Rule -7	Rule -5	Rule -4

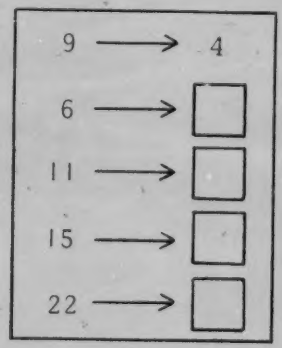
Activity B

Draw this chart on the board:



Have the children study the pair of numbers in the first row of the chart. The same rule will be used for getting each number in the second column. Have several children come to the chalkboard and fill in the missing numerals. Then ask them to identify the rule used in the chart. (+4.)

Now copy the following chart on your chalkboard:



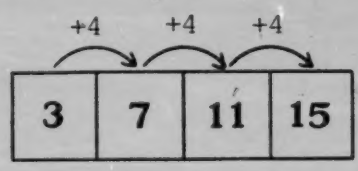
Ask the children what rule they think was used this time. (-5.)

Have different children come up and fill in the missing numerals. Continue with other examples similar to these until you feel the children are ready to complete Worksheet 9. Let the children use their number lines to help find the solutions.

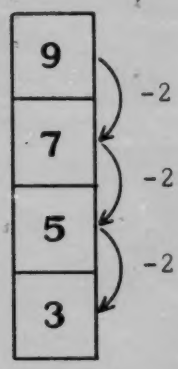
Activity C

Have the children look at Worksheet 10. Explain that this worksheet is like a crossword puzzle, but instead of filling in the boxes with letters, they are to fill in the boxes with numerals. Explain that the same rule (e.g., +2) prevails throughout an entire row or column.

Examples:



or

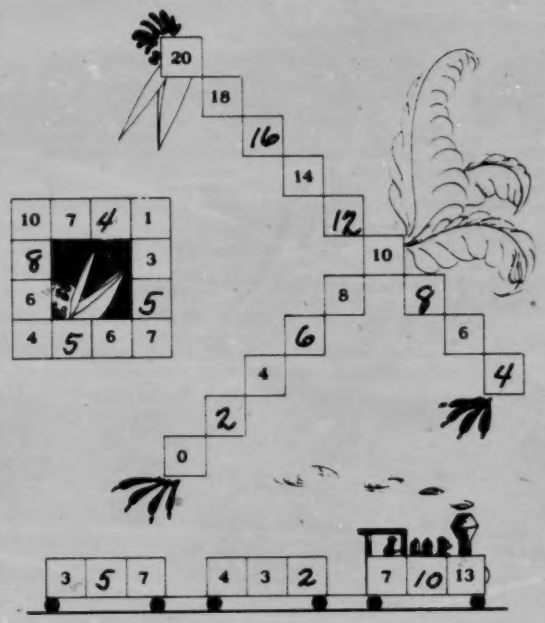


You may have to help the children with the first puzzle. Have them complete the remaining puzzles on the worksheet.

Worksheets 11 and 12 are similar to Worksheet 10, but become increasingly more difficult. They are enrichment activities and should be completed during the students' free time.

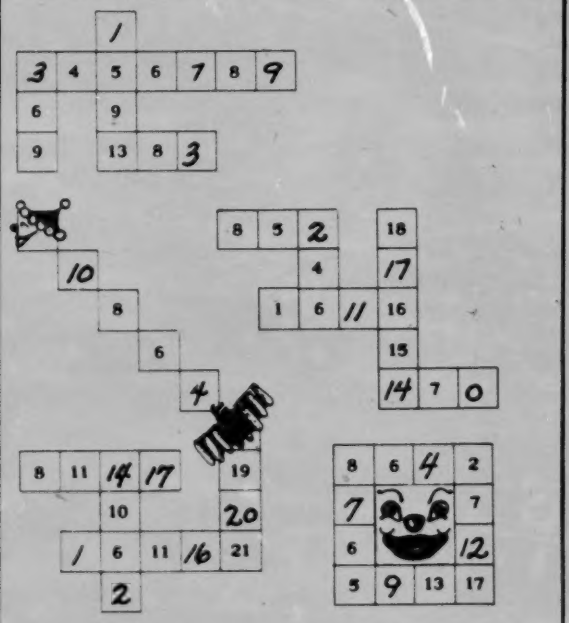
Worksheet 10
Unit 20 Name _____

Find the patterns. Fill in the empty boxes.



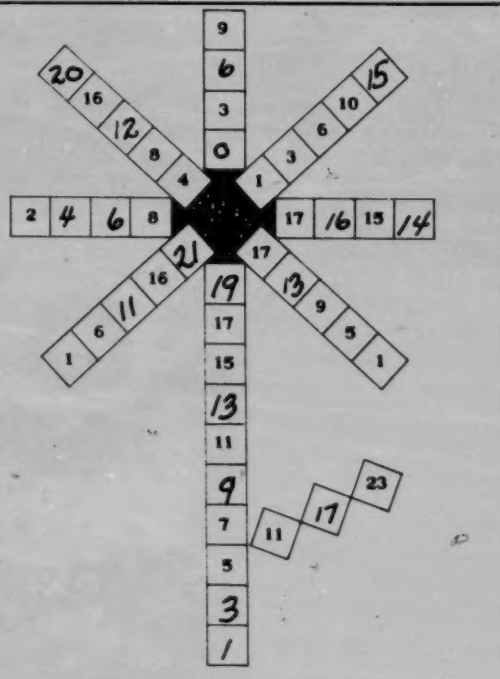
Worksheet 11
Unit 20 Name _____

Each line of numbers has its own pattern.
Find the patterns and fill in the empty boxes.



Worksheet 12
Unit 20 Name _____

Find the patterns. Fill in the empty boxes.



Lesson 4: ASSOCIATIONS

In this lesson the children are introduced to elementary ideas about associations. These concepts will be more explicitly developed in later grades. The activities also illustrate that addition and subtraction are inverse operations.

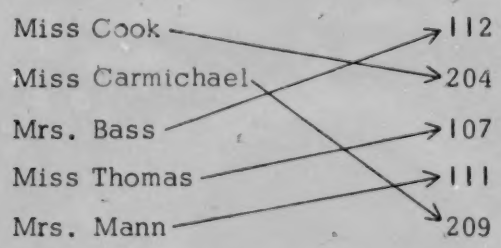
MATERIALS

- Worksheets 13 and 14

PROCEDURE

Activity A

The purpose of this activity is to make the children aware of the associations among objects with which they are familiar. Ask several children to hold up their reading books. Establish the fact that each student knew which book was associated with him. (Perhaps his name was on it.) Suggest other things that are associated with him, e.g., his name, his coat, his teacher. Then write on the chalkboard the names of some teachers in the school. Make a list of the numbers of the rooms that these teachers occupy, but do not list the room numbers in the same order as the teachers' names. Ask the children if the teachers' names are associated with the numbers in any way. You might indicate the associations by drawing arrows from the name to the proper room number.



Finally, make a list of several students' first names and a random list of their last names. Have the children draw arrows from the first name to the corresponding last name.

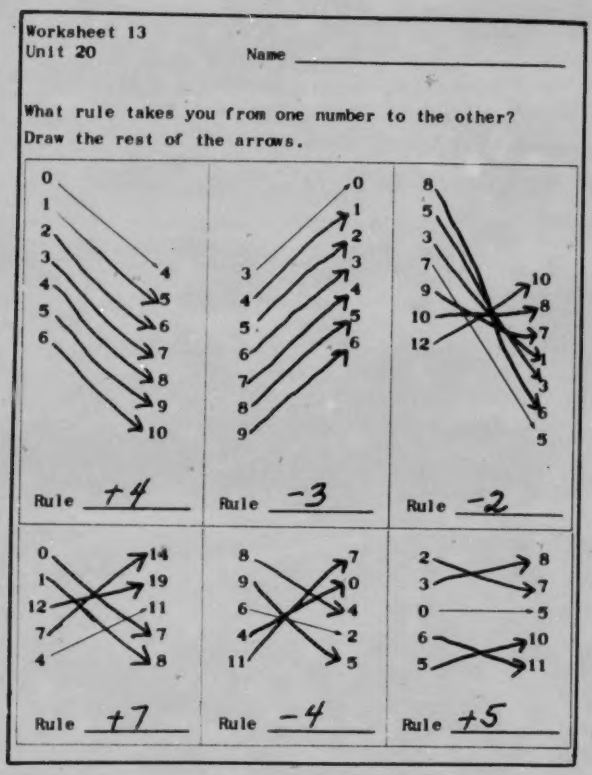
Summarize this activity by indicating that there are many associations between objects; some associations are very clear and others are not so obvious.

Activity B

Write the following columns of numerals on the chalkboard.

9	12
8	9
10	11
6	5
2	13

Tell the children that we would like to associate the numbers in the first column with the numbers in the second column. Tell them that there are many ways these numbers could be associated but we want to consider one special association.

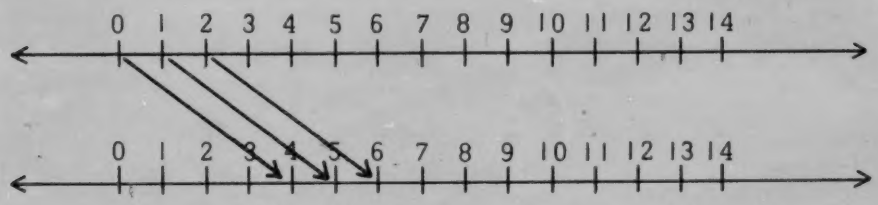


Draw an arrow from the 2 in the first column to the 5 in the second column. Tell the children that this arrow can tell the rule for associating the numbers in the two columns. Ask if anyone can guess the rule the arrow indicates. If they need more help, draw in the arrow associating the 6 and the 9. Have volunteers draw arrows connecting the rest of the pairs according to the same rule. (Each number is associated with a number which is 3 greater.)

When you think the children understand this procedure, have them do Worksheet 13. You may want to work the first example on the worksheet with them.

Activity C

Draw two parallel number lines on the chalkboard.

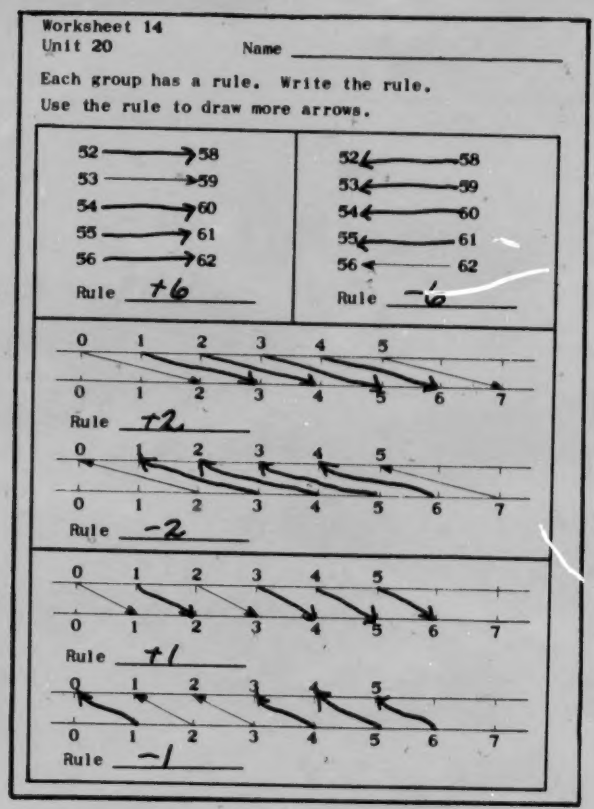


Have the children help you draw the arrows that associate the numbers on the first number line with those numbers that are 4 larger on the second number line. (The rule is "add 4 to the first number to get the second number.") Ask:

DOES SOMEONE KNOW HOW WE COULD SHOW SUBTRACTION OF FOUR WITH THE SAME ARROWS? (Reverse the direction of the arrow heads.)

Point out that to add 4 to a number on the top number line, we follow the arrow to the bottom number line (2 $\longrightarrow$ 6). To subtract 4 from a number on the bottom number line we follow the arrow to the top number line (6 $\longrightarrow$ 2).

Review the fact that subtraction is the inverse of addition. It might be meaningful to say that subtraction "undoes" addition. Have the children do Worksheet 14 which illustrates that addition and subtraction are inverse operations. They should write the rule for each problem on the line below it (add 6, subtract 6, etc.).



Worksheet 14
Unit 20 Name _____

Each group has a rule. Write the rule.
Use the rule to draw more arrows.

52 → 58
53 → 59
54 → 60
55 → 61
56 → 62
Rule +6

52 ← 58
53 ← 59
54 ← 60
55 ← 61
56 ← 62
Rule -6

0 1 2 3 4 5
0 1 2 3 4 5 6 7
Rule +2

0 1 2 3 4 5
0 1 2 3 4 5 6 7
Rule -2

0 1 2 3 4 5
0 1 2 3 4 5 6 7
Rule +1

0 1 2 3 4 5
0 1 2 3 4 5 6 7
Rule -1

Lesson 5: ADDITION AND SUBTRACTION AS INVERSE OPERATIONS

The activities in this lesson give the children more experience with the idea that addition and subtraction are inverse operations. First the children use a number line, making forward and backward moves that undo each other. Then a "machine" is introduced which is used to provide further practice with addition and subtraction.

MATERIALS

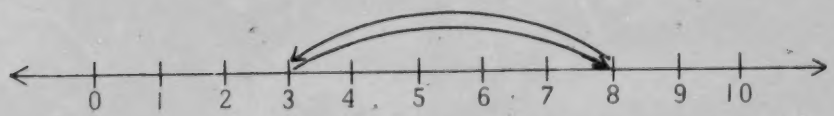
- student number lines (optional)
- counters (optional)
- Minnebars (optional)
- Worksheets 15, 16 and 17

PROCEDURE

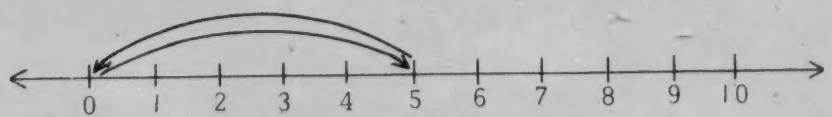
Activity A

Draw a number line on the chalkboard. Have a child use it to help solve a series of problems similar to those below. If your class needs number lines, counters or Minnebars to help visualize these problems, use them.

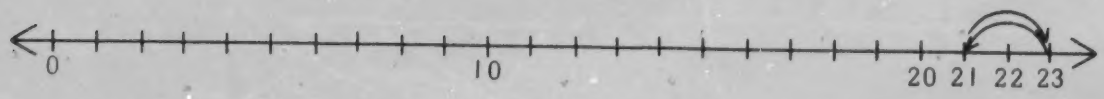
Sam had 3 cents.
He found 5 cents.
He spent 5 cents.
How many cents did he have then?



Jane picked 5 flowers.
She gave away 5 flowers.
How many flowers did she have then?



Bill had 23 marbles.
He lost 2 marbles.
His brother gave him 2 marbles.
How many marbles did he have then?



After one or two problems have been done, discuss the idea that addition and subtraction undo one another.

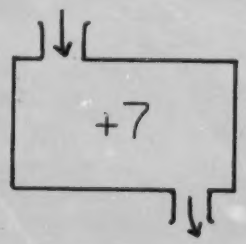
Now write the following problems on the board and have the children help you complete the equations.

$6 - 3 + 3 = \square$

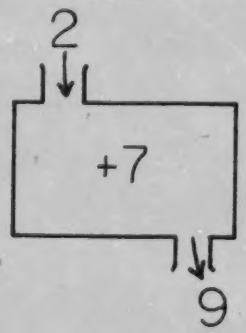
$8 + 2 - 2 = \square$

Activity B

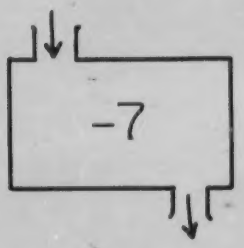
Draw the following diagram on the chalkboard.



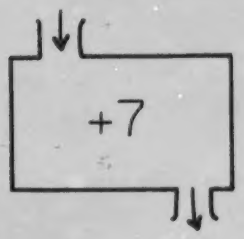
Explain that this is a picture of a machine that adds 7 to any number put into it. Have different children come up and put a few numbers through the machine.

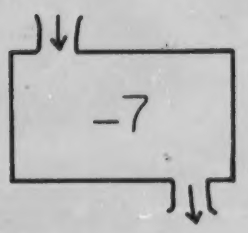


Repeat this procedure with a machine that subtracts 7 from any number put into it.



Now put the two machines together.





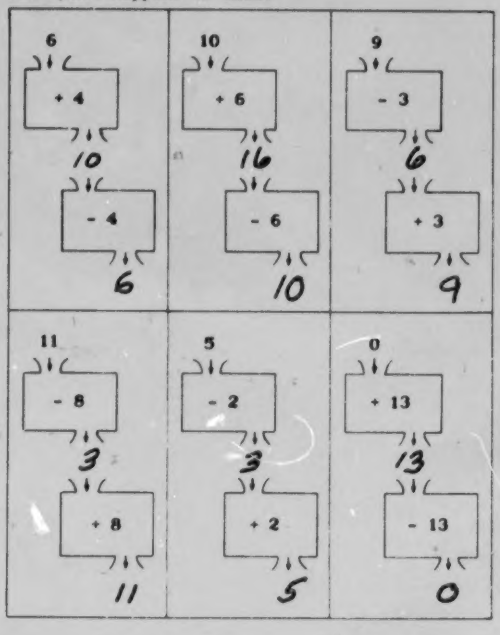
Have a volunteer put a number into the first machine and follow it through the second machine. Ask for an explanation of the results. (The final number is the same as the number that was put into the machine. The second machine undoes the work of the first machine.)

Ask for a volunteer to draw another pair of machines whose joint effect is not to change any number put in it. This will probably result in another add-subtract combination.

Have the children complete Worksheets 15, 16 and 17.

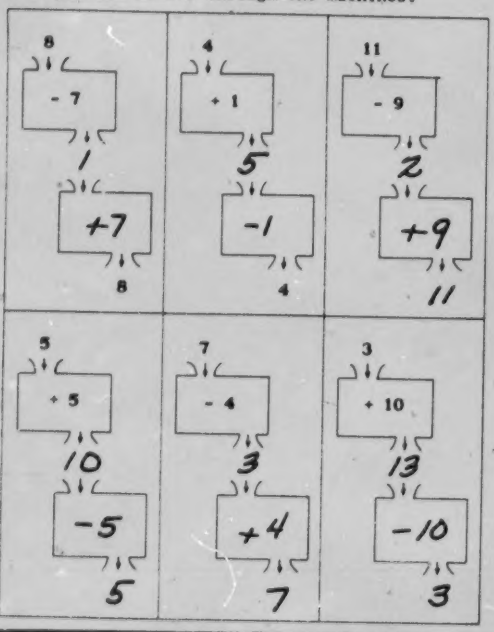
Worksheet 15
Unit 20 Name _____

Feed the numbers into the machines.
Show what happens to them.



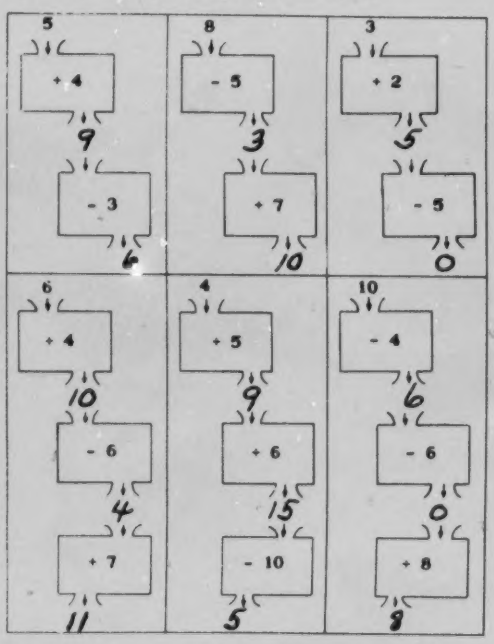
Worksheet 16
Unit 20 Name _____

Decide what machine should be used to undo the first operation.
Then put the number through the machines.



Worksheet 17
Unit 20 Name _____

Put the numbers through the machines.



Lesson 6: PLACE VALUE

This lesson reviews the idea of place value in a base ten numeration system. It provides background for the standard algorithms (rules) for addition and subtraction.

MATERIALS

- $\frac{1}{2}$" grid on tagboard, 3 sheets per student
- envelopes for storing grid pieces, 1 per child
- abacus
- number line tapes from addition slide rules
- Worksheets 18 and 19

PROCEDURE

Activity A

In Unit 16, Numbers and Measuring, the children rewrote three-digit numerals: 365 as 3 hundreds + 6 tens + 5 ones. They also used the name T · T for hundred and T for ten, so they got $365 = 3(T \cdot T) + 6T + 5$.

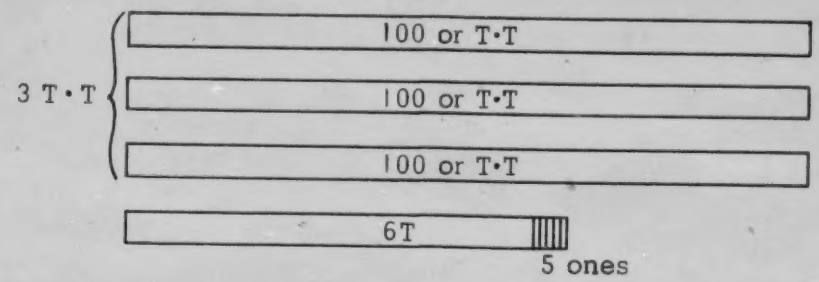
Write a three-digit numeral on the chalkboard and ask the students what each digit in it represents. It might help to write the following sentence on the chalkboard:

365 = ___ hundreds + ___ tens + ___ ones

The following chart is also helpful:

	T · T's	T's	1's
365 =	3	6	5
482 =	4	8	2

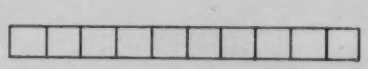
Give the children more three-digit numerals. When you feel they have had enough practice explaining the value of each digit, ask a student to use number line tapes to show what is meant by 365. He can use several number line tapes and attach them to the chalkboard.



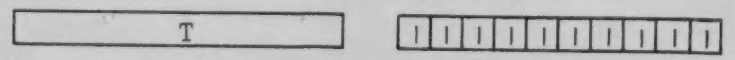
Activity B

Give each child 3 sheets of tagboard on which a $\frac{1}{2}$" grid is printed, and an envelope in which to store the pieces after they have been cut out. (The children could bring large used envelopes from home.) Have each student cut a row of squares from a long edge of a grid. This strip should then be cut into $\frac{1}{2}$" squares.

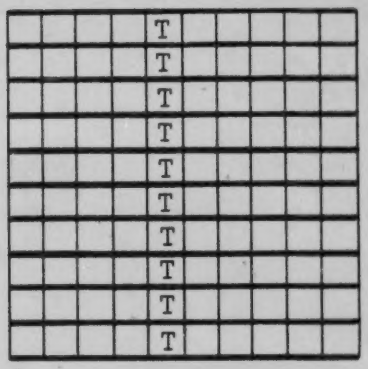
Ask each child to place ten of these small square units side by side.



Then have the children make ten-unit strips by cutting another row from the edge of the grid and cutting off a ten-unit strip. (If a ten-unit strip is placed beside the row of ten squares, the equivalence can be checked.) The unmarked side of each ten-unit strip can be marked T for ten and the other side can be marked with a 1 on each $\frac{1}{2}$" square.



Ask your class how many tens it takes to make a hundred. If they are uncertain about the answer, illustrate a hundred on the board with a 10 x 10 array. Then write one hundred in various ways, e.g., 100, one hundred, 10 x 10 and T · T.



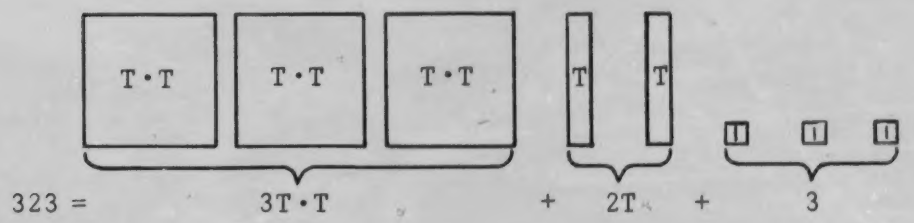
Have each child mark a 10 x 10-unit square on his grid sheet. After checking his markings with you, he should cut out that piece and label it on one side to show that it contains ten rows of ten square units each.

The other side should be marked with a T · T and 100.

Using the T · T piece as a model, each child should cut out as many T · T pieces as possible from his grid sheets. From what remains, he should cut out all the possible T strips. The rest of the paper can be cut into unit pieces. T · T and T should be written on the appropriate pieces.

Activity C

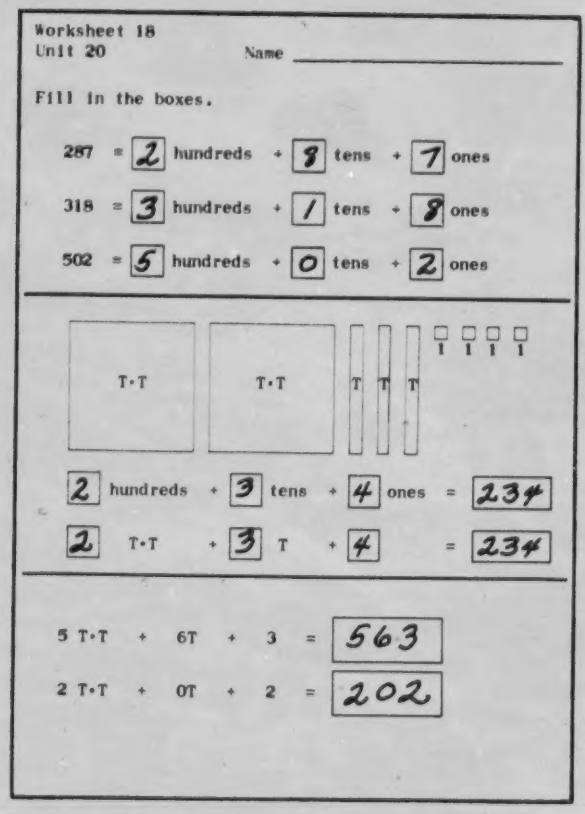
Write a three-digit numeral on the chalkboard. (Keep it under 600, because each child will have only six T · T pieces; some may have only five.) Tell the children to use their grid pieces to represent this number on their desks. For example:



Also show this number with number line tapes and an abacus. Repeat the activity with a few other numbers, e.g., 251, 18, 307, and 50.

Have the children complete Worksheets 18 and 19. Allow them to use their grid pieces when they fill out the worksheets. If they have much difficulty with the worksheets, they will need more practice before going on to Lesson 7.

NOTE: Have the children store their grid pieces in their envelopes. They will be used in Lessons 7 and 8 and 10 through 13.


Worksheet 18
Unit 20 Name _____

Fill in the boxes.

287 = 2 hundreds + 8 tens + 7 ones

318 = 3 hundreds + 1 tens + 8 ones

502 = 5 hundreds + 0 tens + 2 ones

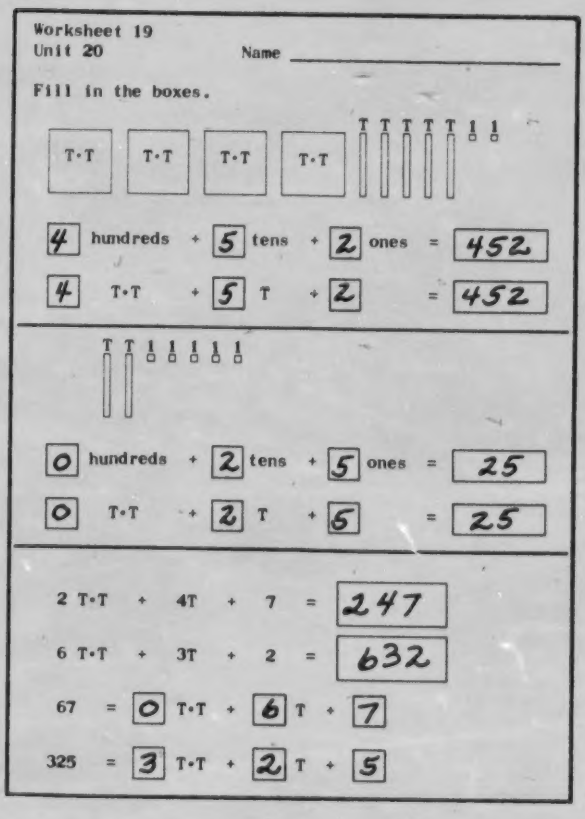
T·T T·T T T T 1 1 1 1

2 hundreds + 3 tens + 4 ones = 234

2 T·T + 3 T + 4 = 234

5 T·T + 6T + 3 = 563

2 T·T + 0T + 2 = 202


Worksheet 19
Unit 20 Name _____

Fill in the boxes.

T·T T·T T·T T·T T T T T T 1 1

4 hundreds + 5 tens + 2 ones = 452

4 T·T + 5 T + 2 = 452

T T 1 1 1 1 1

0 hundreds + 2 tens + 5 ones = 25

0 T·T + 2 T + 5 = 25

2 T·T + 4T + 7 = 247

6 T·T + 3T + 2 = 632

67 = 0 T·T + 6 T + 7

325 = 3 T·T + 2 T + 5

Lesson 7: HOW MANY NUMBER FACTS DO WE NEED?

The children add and subtract pairs of numbers that are multiples of ten (e.g., 30 + 40) and also pairs of numbers where one number has two digits and the other number has one digit (e.g., 54 + 2). They will see that because of the place value system, only the basic one-digit number combinations need to be used to find the answers.

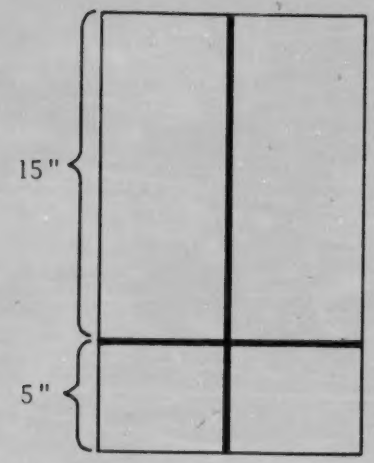
Beginning with this lesson, you will be doing a good deal of work with place value charts. Depending on the materials you have available, you may prefer to use the flannel board, the chalkboard, or a combination of the two. You will need felt numerals if you use the flannel board exclusively. Use yarn or thin felt strips to make the chart divisions on the flannel board. The lessons have been written for using both the flannel board and the chalkboard, but you may vary this quite easily if you wish.

MATERIALS

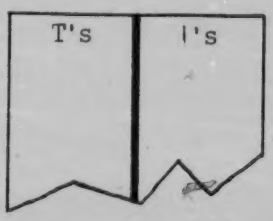
- grid pieces (from Lesson 6)
- flannel board and felt strips (optional)
- sheets of 12" x 20" construction paper or newsprint
- masking tape
- Worksheets 20, 21 and 22

PREPARATION

For this and future lessons in this section, the children will also need place value charts of their own. If you want to let them make the charts themselves, give each child a sheet of 12" x 20" construction paper or a sheet of newsprint. Have them draw a heavy line across the sheet, 5 inches from the bottom, and another heavy line down the middle of the sheet. Put a sketch on the board for them to follow.



Show the children how to label the chart like this:



Another way to make the place value chart is to use masking tape to divide each child's desk top into sections. The children could write the labels on note cards and tape the cards in place.

PROCEDURE

Activity A

Write the following incomplete sentences on the chalkboard and ask for volunteers to complete them.

20 = ☐ tens + ☐ ones = ☐ T + ☐ = ☐ T

30 = ☐ tens + ☐ ones = ☐ T + ☐ = ☐ T

2T IS ANOTHER WAY TO REPRESENT 20. 3T IS ANOTHER WAY TO REPRESENT 30. WHAT IS ANOTHER WAY TO REPRESENT 50? (5T.)

Write the following on the chalkboard.

$$20 + 30 = 2T + 3T = 5T = 50$$

WE COULD ALSO WRITE IT THIS WAY:

$$\begin{array}{r} 20 \\ +30 \\ \hline 50 \end{array}$$ (Put this on the chalkboard, also.)

Work several examples similar to the following on the chalkboard.

$$40 + 30 = 4T + 3T = 7T = 70 \qquad \begin{array}{r} 40 \\ +\ 30 \\ \hline 70 \end{array}$$

Similarly, show that:

$$50 - 20 = 5T - 2T = 3T = 30 \qquad \begin{array}{r} 50 \\ -\ 20 \\ \hline 30 \end{array}$$

Then have the children do Worksheet 20.

Worksheet 20
Unit 20 Name _____

Finish each equation.

40 + 50 = [4] T + [5] T = [9] T = [90]

20 + 50 = [2] T + [5] T = [7] T = [70]

70 + 10 = [7] T + [1] T = [8] T = [80]

60 + 20 = 6T + 2T = 8T = 80

80 + 10 = 90

30 + 50 = 80

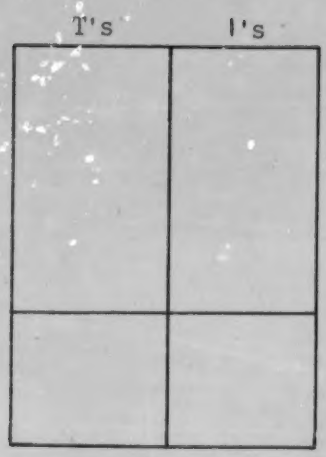
40 - 10 = [4] T - [1] T = [3] T = [30]

70 - 20 = 7T - 2T = 5T = 50

80 - 30 = 50

$$\begin{array}{r} 60 \\ -\ 50 \\ \hline 10 \end{array} \qquad \begin{array}{r} 30 \\ +\ 40 \\ \hline 70 \end{array} \qquad \begin{array}{r} 90 \\ -\ 50 \\ \hline 40 \end{array}$$

Activity B

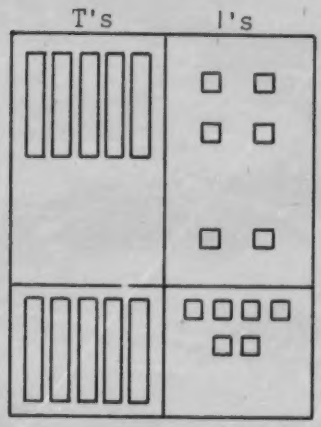


54

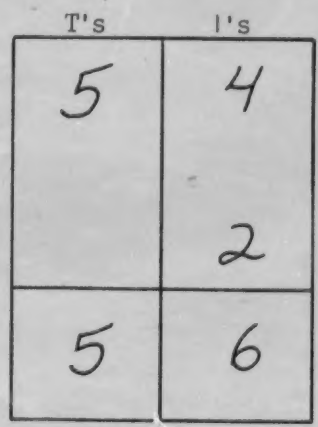
Write $\underline{+2}$ on the chalkboard. Make two charts, similar to the one to the left, on the flannel board and/or chalkboard.

Suggest that the grid pieces from Lesson 6 could be used on the flannel board to show this problem. Have volunteers represent the numbers with the appropriate grid pieces. (Put a piece of masking tape on the back of the grid pieces.) Then have another volunteer write in the corresponding numbers on the chalkboard chart. The rest of the children can work with their own grid pieces at their desks.

$$\begin{array}{r} 54 \\ \underline{+2} \\ 56 \end{array}$$



flannel board chart



chalkboard chart

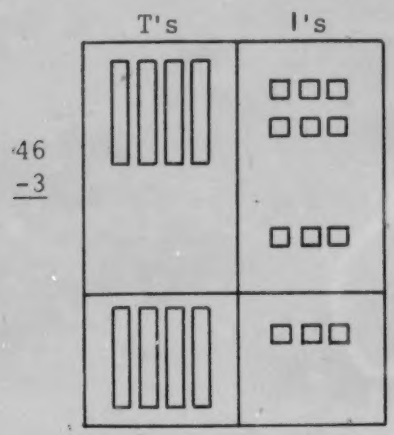
Point out that the following way of writing the problem in T-notation is very much like the notation on the chalkboard chart.

$$\begin{array}{r} 5T + 4 \\ \underline{\quad + 2} \\ 5T + 6 = 56 \end{array}$$

Repeat this procedure with several more addition problems.

Activity C

Develop subtraction in a way similar to the way addition was developed in Activity B.



T's	1's
4	6
	3
4	3

Worksheet 21
Unit 20 Name _____

Complete the addition chart.

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

50

Activity D

Have the children complete the addition chart on Worksheet 21. On the chalkboard write several addition combinations of the kind that you have already considered in this lesson.

20	70	25	62
+40	+20	+3	+7
60	90	28	69

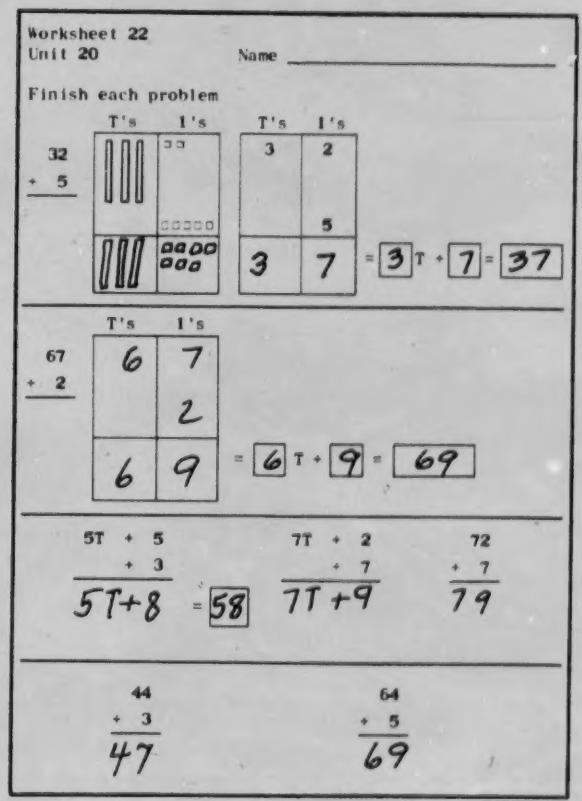
ARE THESE NUMBER COMBINATIONS ON YOUR ADDITION CHART ON WORKSHEET 21? (No.)

DO YOU THINK THE CHART NEEDS TO BE MADE LARGER TO SHOW NUMBERS AS BIG AS THESE?

These questions should begin a discussion that leads to the following conclusions: Only one-digit number combinations need to be used to add the numbers in these problems. The addition chart does not have to be made larger to be helpful with these problems.

Worksheet 22 can now be completed by the children.

NOTE: Have the children save their place value charts for use in Lessons 8, 10, 11 and 12.



Worksheet 22
Unit 20 Name _____

Finish each problem

32 + 5 = 37 (3 T + 7 = 37)

67 + 2 = 69 (6 T + 9 = 69)

5T + 5 + 3 = 5T+8 = 58

7T + 2 + 7 = 7T+9

72 + 7 = 79

44 + 3 = 47

64 + 5 = 69

Lesson 8: ADDITION FACTS AND PLACE VALUE

In this lesson, the algorithms (rules) are developed for adding and subtracting two-digit numbers where regrouping is not needed, e.g.,

$$\begin{array}{r} 53 \\ +\,15 \\ \hline 68 \end{array} \qquad \begin{array}{r} 43 \\ -\,12 \\ \hline 31 \end{array}$$

MATERIALS

- grid pieces (from Lesson 6)
- student place value charts (from Lesson 7)
- blank paper
- masking tape
- Worksheets 23 through 26

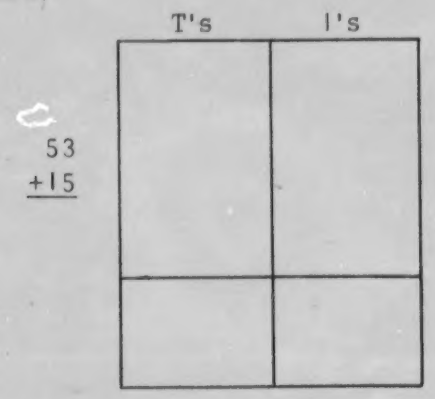
PROCEDURE

Activity A

Give the children their grid pieces and a sheet of paper on which to record problems.

Make two charts, similar to the one below, on the flannel board and/or chalkboard. Write the problem 53 on the chalkboard.

$$\begin{array}{r} 53 \\ +\,15 \\ \hline \end{array}$$



Have the children use their grid pieces to show the problem on their own place value charts. Ask for a volunteer to put

grid pieces on the flannel board chart with masking tape and to record the corresponding numbers on the chalkboard chart.

Explain that the charts show the problems clearly, but they are bothersome to make. There are other, shorter ways to show the same problem. Put these examples on the chalkboard:

5T + 3	50 + 3	53
+ T + 5	+10 + 5	+15
6T + 8	60 + 8	8
		60
		68

Be sure that the children understand where the 8 and the 60 come from in the right-hand version. You may want to write it on the board as follows:

53	
+15	
8	8 = 3 + 5
+60	60 = 50 + 10
68	

By questioning the class, bring out the fact that they only need to know the single-digit addition facts to do this addition, since the ones and tens places are added separately. It is all right for the children to use any form they wish.

Work the following subtraction problem the same way you worked the addition problem.

43
-12

Tell the class:

SINCE SUBTRACTING 12 IS THE SAME AS SUBTRACTING

10 AND THEN SUBTRACTING 2, ANOTHER FORM OF WRITING THE PROBLEM IS:

$$\begin{array}{r} 4T + 3 \\ -\ 1T - 2 \\ \hline \end{array}$$

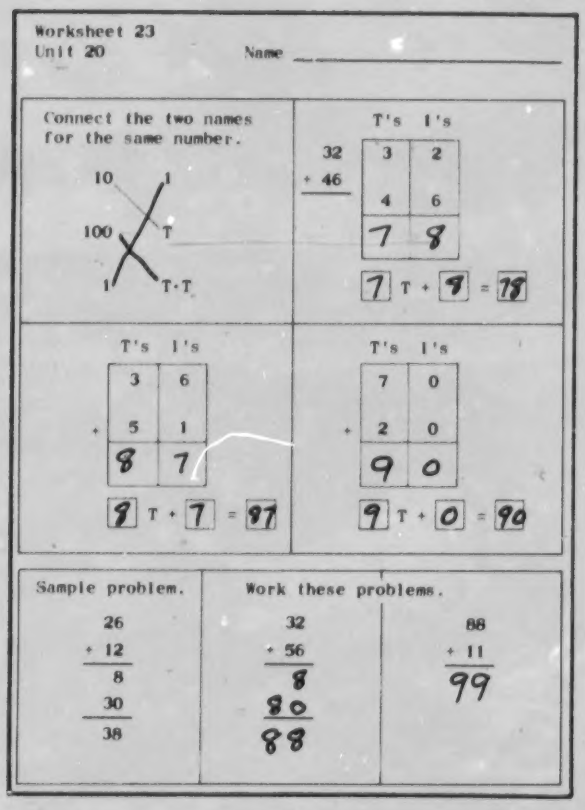
(Put this on the chalkboard.)

Repeat this procedure for as many other addition and subtraction problems as you feel are necessary. None should require regrouping. Some examples are:

21	57	82	23
+32	−24	+13	−12
___	___	___	___

Activity B

Have the children complete Worksheets 23, 24, 25 and 26. Then have them make up story problems to go with addition and subtraction combinations that you write on the board. None of the combinations should require regrouping.



Worksheet 23
Unit 20 Name _____

Connect the two names for the same number.

10 — 1
100 — T
1 — T·T

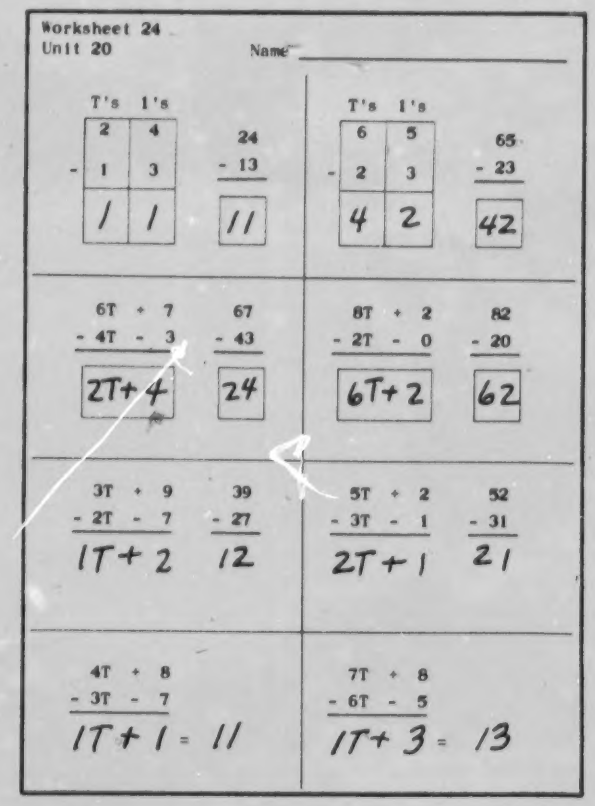
T's 1's
32 + 46 = 3 2 / 4 6 = 7 8
7 T + 8 = 78

T's 1's
3 6 + 5 1 = 8 7
8 T + 7 = 87

T's 1's
7 0 + 2 0 = 9 0
9 T + 0 = 90

Sample problem.
26 + 12 = 8 + 30 = 38

Work these problems.
32 + 56 = 8 + 80 = 88
88 + 11 = 99



Worksheet 24
Unit 20 Name _____

T's 1's
2 4 − 1 3 = 1 1
24 − 13 = 11

T's 1's
6 5 − 2 3 = 4 2
65 − 23 = 42

6T + 7 − 4T − 3 = 2T + 4
67 − 43 = 24

8T + 2 − 2T − 0 = 6T + 2
82 − 20 = 62

3T + 9 − 2T − 7 = 1T + 2
39 − 27 = 12

5T + 2 − 3T − 1 = 2T + 1
52 − 31 = 21

4T + 8 − 3T − 7 = 1T + 1 = 11

7T + 8 − 6T − 5 = 1T + 3 = 13

Worksheet 25
Unit 20 Name _____

1. There are 76 children in the second grade classes.
They are going to take a field trip to the pond.
The first bus will hold 45 children.
How many will go on the next bus?

$$\begin{array}{r} 76 \\ -45 \\ \hline 31 \end{array}$$

2. There were 37 mallard ducks sitting on the pond when the class arrived.
15 ducks flew away.
How many were left?

$$\begin{array}{r} 37 \\ -15 \\ \hline 22 \end{array}$$

3. The children saw 3 mother ducks with their families.
There were 4 ducklings in each family.
How many ducklings did the children see?

$4+4+4=12$ or $3 \cdot 4 = 12$

Worksheet 26
Unit 20 Name _____

1. There were 34 mallard ducks on the pond.
45 more flew in at sunset and sat on the pond.
How many ducks were there altogether?

$$\begin{array}{r} 34 \\ +45 \\ \hline 79 \end{array}$$

2. 87 mallards stopped overnight at the pond.
At dawn 42 flew away.
How many stayed behind?

$$\begin{array}{r} 87 \\ -42 \\ \hline 45 \end{array}$$

3. An hour later 22 more ducks flew away.
How many flew away altogether?

$$\begin{array}{r} 42 \\ +22 \\ \hline 64 \end{array}$$

Lesson 9: WORD PROBLEMS

Word problem cards are used in this lesson to provide addition and subtraction practice. The numbers in the addition and subtraction combinations have been omitted so the problems can be varied to provide the type of practice needed by the children. Although the cards are first used in this lesson, they should be used during free time or whenever you consider them appropriate or necessary during the rest of the school year. You can make the addition and subtraction combinations increasingly more difficult as the children learn to cope with them.

MATERIALS

- decks of word problem cards (printed materials)
- spinners and dials (from Section I)
- dice (from Section I)
- counters (optional)
- Minnebars (optional)
- addition slide rules (optional)

PROCEDURE

Each deck of cards contains 24 word problems from which the numbers have been omitted. The children supply the required numbers and solve the problems. Some possible ways to generate numbers are suggested below, but also use ways the children might devise.

1. Each child prepares about twenty slips of paper or counters by marking each one with a numeral from 1 to 10. (Later, larger numbers can be used.) These are placed in a box and mixed. Then the numbers for a particular problem are selected by drawing two of the marked pieces.
2. Make up lists of numeral pairs. This way you can control the combinations and provide numerals appropriate to the skills of different pairs of children. The numbers for a problem are taken in sequence from one of these lists. If a subtraction problem is being considered, help the children see that it is necessary to order the numeral pairs

so that the lesser number is subtracted from the greater number. This necessity is illustrated by the problem below.

> Mrs. Wren baked ____ cupcakes for a party. If ____ were eaten, how many were left?

3. Use the game spinners from Game 3 in Section I and mark them with appropriate numbers. Each child in a pair spins once to obtain the two numerals for a problem.
4. Throw a pair of dice to obtain the numerals. Use two of the positive dice from Game I in Section I.

The class may separate into partners to use the cards. Each pair of partners gets a set of card sheets. Have the children cut each sheet into four cards. They choose a word problem card and obtain two numbers for it. Each partner writes a complete equation to solve the problem and compares it with his partner's result. Minnebars, counters, or addition slide rules may be used to illustrate a problem.

You may wish to spot check the results as the children play with the cards, or perhaps pairs of children can compare results and check those that differ.

Lesson 10: ESTIMATING SUMS

This lesson introduces the idea of approximating a sum to check whether or not an answer to an addition problem is reasonable. More addition and subtraction practice is also provided.

It is often helpful to be able to make an estimate that will show whether the calculated answer to a problem is reasonable. For example, is the answer below reasonable?

$$78 + 85 = 1513$$

No, because the sum should be somewhere between 150 and 170. To show this we can sandwich the addends between the nearest tens.

$$70 < 78 < 80$$

$$80 < 85 < 90$$

(Remember that $70 < 78 < 80$ can be read as "70 is less than 78 and 78 is less than 80," or "78 is between 70 and 80.") Then we can add the upper estimates of each of the addends to get the upper estimate of the sum, and we can add the lower estimates of the addends to get the lower estimates of the sum.

$$70 + 80 < 78 + 85 < 80 + 90$$

In the lower estimate (70 + 80) each addend is less than the addends of the original sum (78 + 85). In the upper estimate (80 + 90) each addend is more than the addends of the original sum. So we have estimated the sum 78 + 85 by sandwiching it between two other sums that are easier to compute. We have found that the sum lies between 150 and 170.

$$150 < 78 + 85 < 170$$

This is the idea that will be presented to the children in this lesson. If some child asks about using this method for estimating differences, explain only that it is much more difficult and we won't be doing it in second grade.

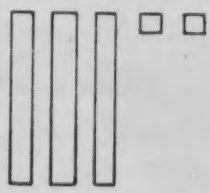
MATERIALS

- grid pieces (from Lesson 6)
- student place value charts (from Lesson 7)
- flannelboard and felt numerals (optional)
- masking tape
- Worksheets 27 through 30

PROCEDURE

Activity A

If you have felt numerals you may prefer to use the flannel board for this activity, otherwise use masking tape to put three T-pieces and two I-pieces on the chalkboard. Have a student write the corresponding numeral underneath.



32

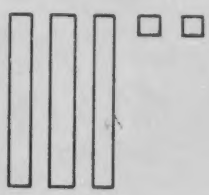
WHAT NUMBER WILL BE SHOWN IF THE TWO I-PIECES ARE REMOVED? (30.)

Write this to the left of the 32 and use three T-pieces to show it.

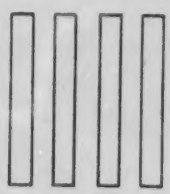
WHAT NUMBER WILL BE SHOWN IF THE TWO I-PIECES ARE EXCHANGED FOR ONE 10-PIECE? (40.)

Write the numeral and use the grid pieces to show this to the right of the 32.

30
3T



32

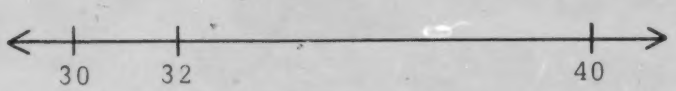


40
4T

Directly beneath the 30 and the 40, write 3T and 4T. Tell the children:

> 30 IS LESS THAN 32, BUT CLOSE TO IT. 3T IS THE CLOSEST WHOLE NUMBER OF T's LESS THAN 32. 40 IS GREATER THAN 32 BUT CLOSE TO IT. 4T IS THE CLOSEST WHOLE NUMBER OF T's GREATER THAN 32. WE CALL 30 THE LOWER ESTIMATE OF 32 AND 40 THE UPPER ESTIMATE OF 32.

Have a child put in the inequality signs (<) showing the ordering: $30 < 32 < 40$. Have another child sketch the appropriate section of the number line.



Repeat this part of the lesson using T-pieces and I-pieces on the board as often as you feel is needed before having the class complete Worksheet 27.

Worksheet 27
Unit 20 Name _____

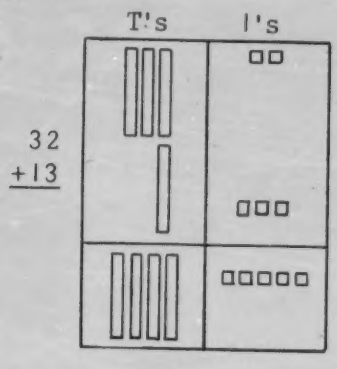
Complete the following problems.
The first one has been done for you.

Lower estimate			Upper estimate
50	< 56 <	60	
80	< 87 <	90	
40	< 42 <	50	
70	< 73 <	80	
20	< 21 <	30	
0	< 6 <	10	
30	< 34 <	40	

Lower estimate			Upper estimate
10	< 17 <	20	
20	< 28 <	30	
30	< 39 <	40	
50	< 51 <	60	
10	< 15 <	20	
10	< 16 <	20	
80	< 89 <	90	

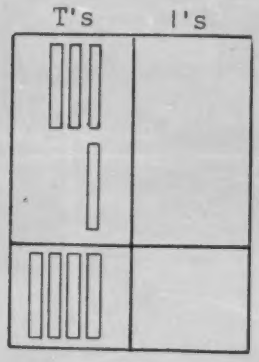
Activity B

Use the grid pieces and masking tape to show an addition problem such as $\begin{array}{r} 32 \\ +13 \\ \hline \end{array}$ on the chalkboard. The children may follow the demonstration by manipulating their grid pieces on their place value charts.

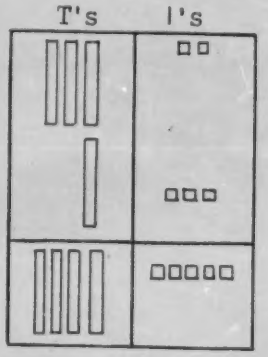


Next sketch charts on the chalkboard showing the addends sandwiched between the nearest tens. Point out that in the left-hand chart the 1-pieces have been removed, so the addends are less than 32 and 13; however, in the right-hand chart, the 1-pieces have been exchanged for T-pieces, so the addends are greater than 32 and 13. The sums on the left should be less than, and the sums on the right should be greater than 32 + 13.

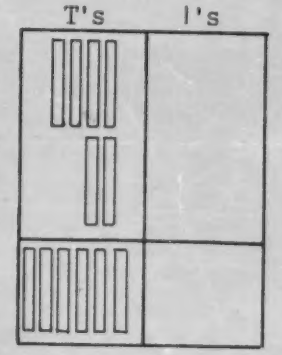
Have a volunteer write the sums underneath the charts. Now the children can look at the three sums and see if the answer they got for the middle one is reasonable.



$\begin{array}{r} 30 \\ +10 \\ \hline 40 \end{array}$



$\begin{array}{r} 32 \\ +13 \\ \hline 45 \end{array}$



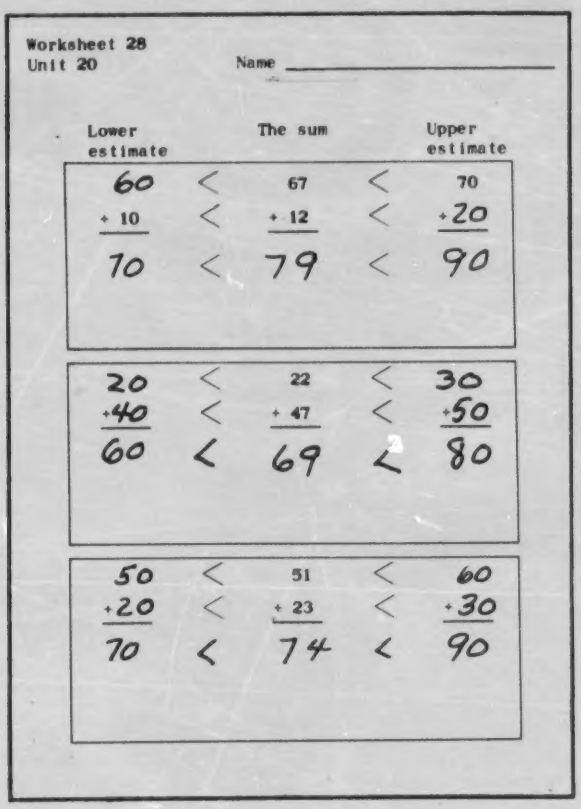
$\begin{array}{r} 40 \\ +20 \\ \hline 60 \end{array}$

After as many demonstrations similar to the above as you feel are necessary, show how an estimate can be made without the charts. For example:

50	<	54	<	60
+20	<	+21	<	+30
70				90

We expect the sum 54 + 21 to be between 70 and 90. Have a child do the computation.

The children can practice making estimates by working in groups of three. One child can round off the addends to the next lowest ten, another child can round off the addends to the next highest ten, and the third child can compute the original sum. Then Worksheet 28 can be used both for practice and evaluation.



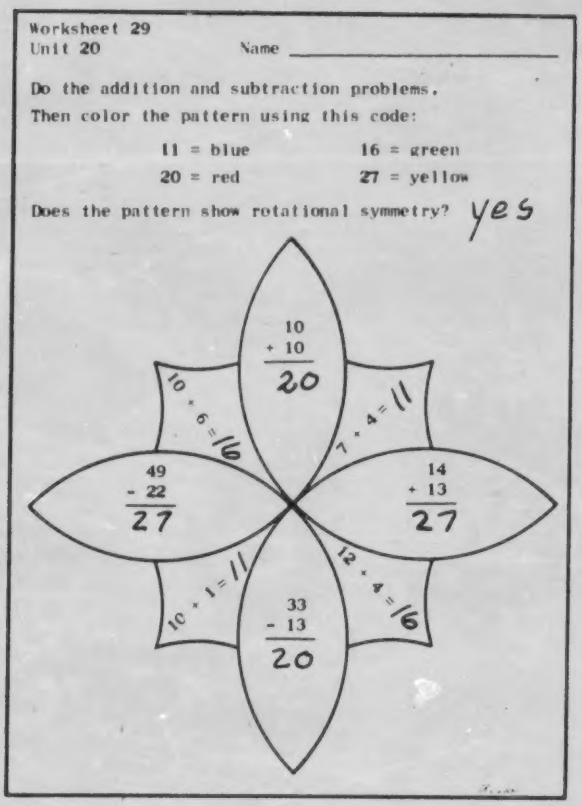
Worksheet 28
Unit 20

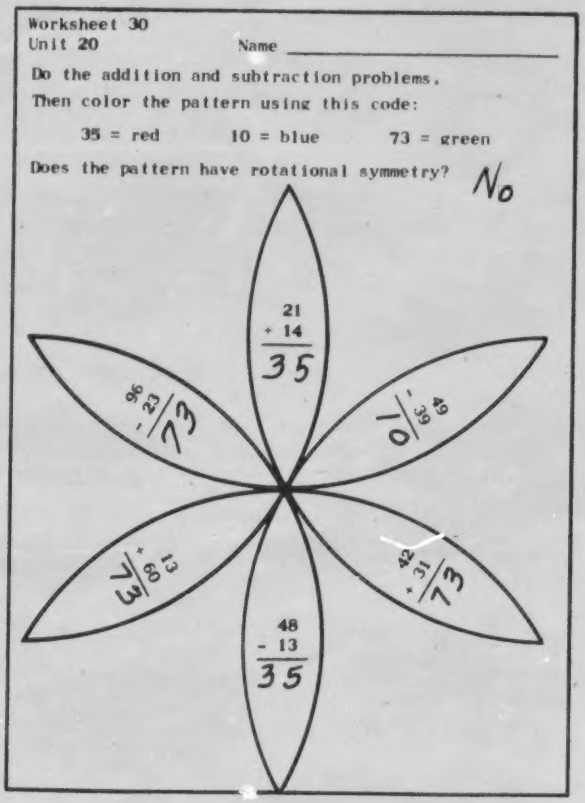
Name _____

Lower estimate		The sum		Upper estimate
60	<	67	<	70
+ 10	<	+ 12	<	+20
70	<	79	<	90
20	<	22	<	30
+40	<	+ 47	<	+50
60	<	69	<	80
50	<	51	<	60
+20	<	+ 23	<	+30
70	<	74	<	90

62

Worksheets 29 and 30 provide more practice in adding and subtracting without regrouping. They also give a short review of rotational symmetry, which was taught in Units 7 and 14. If any of your class are not familiar with this symmetry concept, tell them that a rotationally symmetric pattern is one that looks exactly the same after it has been rotated part of a whole turn. The pattern on Worksheet 29 will be symmetric when the children color it correctly, but the one on Worksheet 30 will not.


Worksheet 29
Unit 20
Name
Do the addition and subtraction problems.
Then color the pattern using this code:
11 = blue
16 = green
20 = red
27 = yellow
Does the pattern show rotational symmetry? yes
10 + 10 = 20
10 + 6 = 16
7 + 4 = 11
49 - 22 = 27
14 + 13 = 27
10 + 1 = 11
12 + 4 = 16
33 - 13 = 20


Worksheet 30
Unit 20
Name
Do the addition and subtraction problems.
Then color the pattern using this code:
35 = red
10 = blue
73 = green
Does the pattern have rotational symmetry? No
21 + 14 = 35
96 - 23 = 73
49 - 39 = 10
57 + 18 = 75
42 + 31 = 73
48 - 13 = 35

Lesson 11: ADDITION WITH REGROUPING

In this lesson the algorithm is developed for addition where regrouping is required. The children had some experience with regrouping on the abacus in Unit 16. Here the grid pieces from Lesson 6 are used for a concrete representation of the regrouping process. The children learn several ways to record what they have done, each method briefer than the previous one. The briefest of these is the standard vertical method:

$$\begin{array}{r} {\scriptstyle 1} \\ 14 \\ +\,19 \\ \hline 33 \end{array}$$

It is not expected that all the children will learn the more condensed forms. Your class will no doubt include children who will continue to use grid pieces or the abacus, as well as those who will learn to think through the steps quickly and write the barest minimum on paper.

MATERIALS

- flannel board and felt numerals (optional)
- masking tape

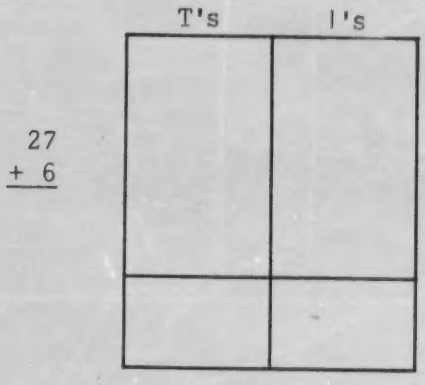
-- for each child --

- grid pieces (from Lesson 6)
- place value chart (from Lesson 7)
- blank paper
- addition slide rule
- abacus (optional)
- Worksheets 31 through 35

PROCEDURE

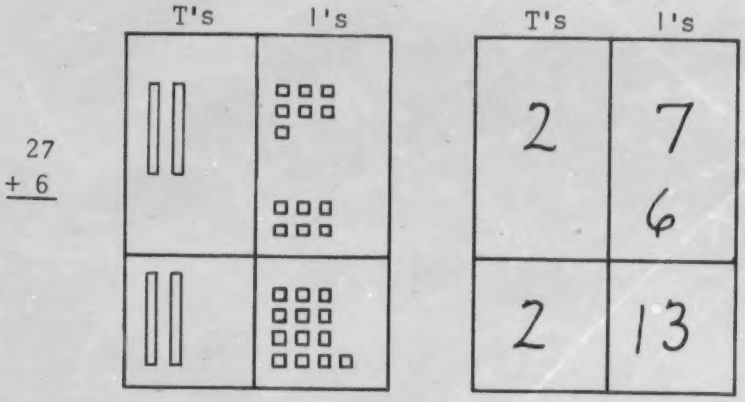
Activity A

Each child should have his 1-pieces and T-pieces from Lesson 6, his place value chart from Lesson 7, and paper for recording problems. Make two charts, like the one below, on the chalkboard and/or flannel board. (Make the upper boxes at least 15 inches tall to allow enough room for three rows of grid pieces.) Write the problem $\begin{array}{r} 27 \\ +\ 6 \\ \hline \end{array}$ on the chalkboard.



Ask one child to show this problem on the first chart, using his grid pieces and masking tape. The other children should show the problem on their individual charts. The result should be similar to the first chart below.

Then ask another child to record the number of members of each set in the second chart. Suggest that he write the first row of numerals near the middle of the upper box.



Call attention to the 13 pieces in the ones column of the first chart.

SHOULD WE HAVE SO MANY 1-PIECES IN THE ONES COLUMN? (No.)

WHAT CAN WE DO WITH ALL THESE 1-PIECES? (Trade ten 1-pieces for a 10-piece.)

Have the child who worked with the pieces make this exchange. Ask the class where we should put the 10-piece. (In the tens column.) Suggest that it is easier to keep track of which pieces represent the original sum and which pieces came from an exchange if that piece is put above the others. (See diagram below.) Then call attention to the second chart.

SHOULD WE HAVE TENS IN THE ONES COLUMN? (No.)

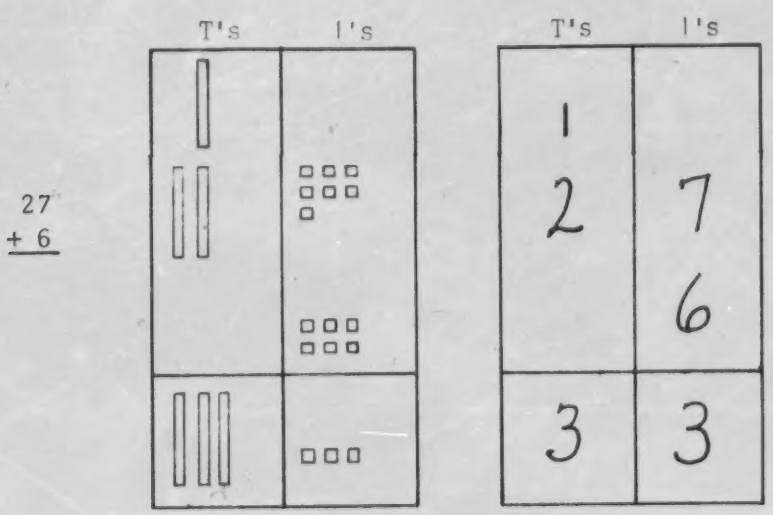
WHAT IS ANOTHER NAME FOR 13? (T + 3.)

WHEN WE TRADE TEN ONES FOR A TEN, WHAT DO WE HAVE LEFT IN THE ONES COLUMN? (3.)

Suggest that it will be least confusing if the numeral representing the ten that was traded for ten ones is written above the other numerals that represent the problem.

NOW HOW MANY TENS DO WE HAVE IN THE TENS COLUMN? (3.)

Now the charts should look like this:

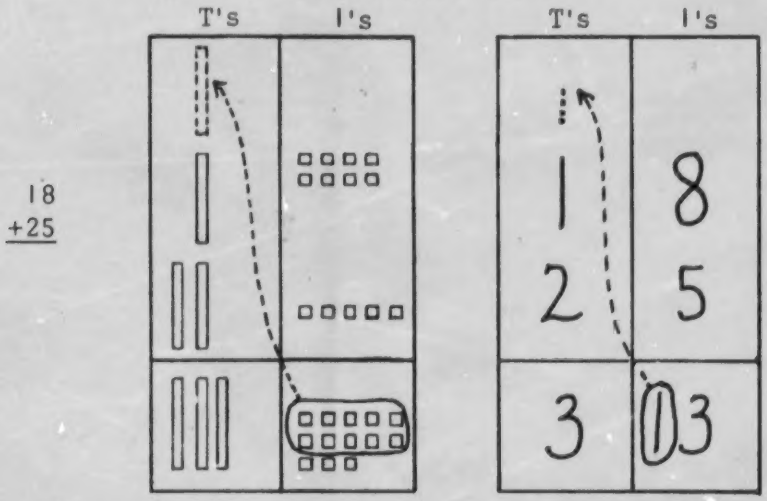


27
+ 6

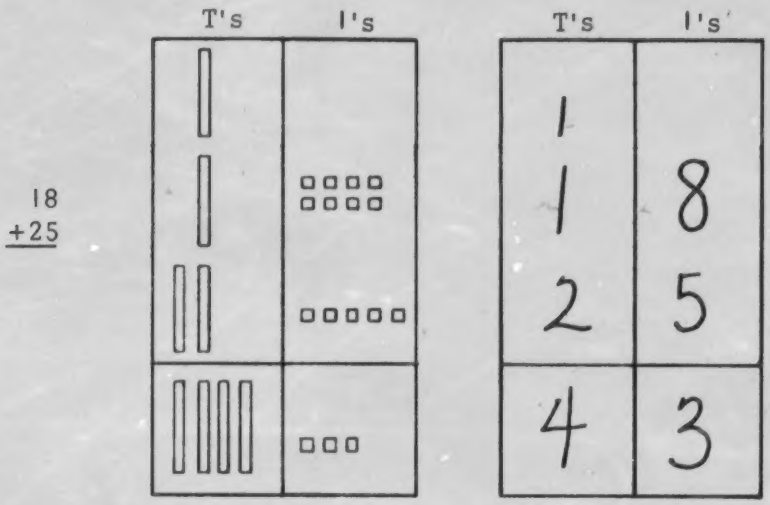
Have the children do the same problem on their addition slide rules and note that they get the same answer. Go through this same procedure using several examples. Have the children work both with grid pieces and with numerals. Some children may need much practice at this stage, while others will be ready to go on to shorter methods of recording the process.

Activity B

Repeat Activity A with two-digit numerals for both addends. Use these forms of recording your procedure.



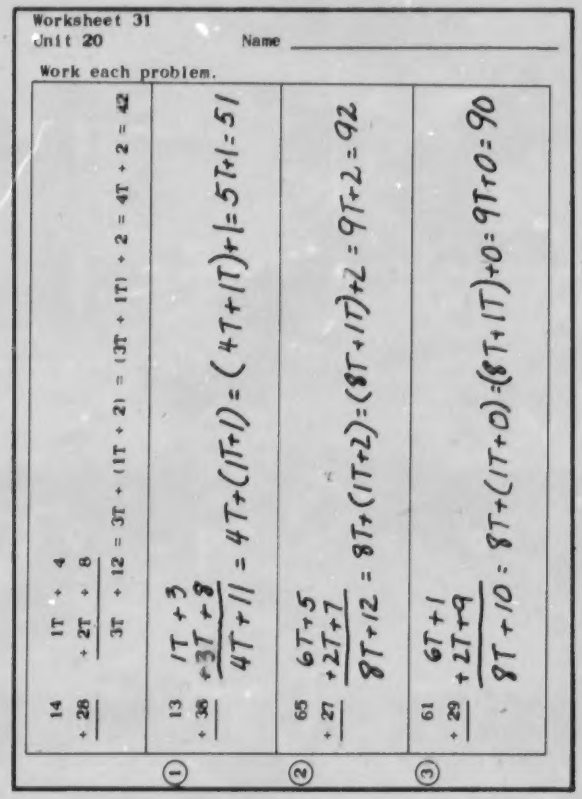
The charts should finally look like this:



Suggest that the same problem can be recorded without bothering with the charts. Keep the charts on the board while you do the problem as shown below.

$$\begin{array}{rcl} 18 & = & 1T + 8 \\ +25 & = & 2T + 5 \\ \hline & & 3T + 13 = 3T + (T + 3) = (3T + T) + 3 = 4T + 3 = 43 \end{array}$$

The children should find it easy to relate this form of notation to the work they have been doing with the charts. When you feel that they are ready, have them do Worksheet 31.



Worksheet 31
Unit 20 Name _____

Work each problem.

14 $\quad 1T + 4$
$+26 \quad +2T + 8$
$3T + 12 = 3T + (1T + 2) = (3T + 1T) + 2 = 4T + 2 = 42$

13 $\quad 1T + 3$
$+38 \quad +3T + 8$
$4T + 11 = 4T + (1T + 1) = (4T + 1T) + 1 = 5T + 1 = 51$

65 $\quad 6T + 5$
$+27 \quad +2T + 7$
$8T + 12 = 8T + (1T + 2) = (8T + 1T) + 2 = 9T + 2 = 92$

61 $\quad 6T + 1$
$+29 \quad +2T + 9$
$8T + 10 = 8T + (1T + 0) = (8T + 1T) + 0 = 9T + 0 = 90$

① ② ③

Activity C

Show these more condensed forms of recording addition to the children, developing them step by step. Do not go on from one form to the next until the children begin to understand the earlier one, but do not expect proficiency at this

time. Have the longer form used in Activity B on the board at the same time, so that the children can follow each step and notice where the short cuts lie.

Form 1:

$$\begin{array}{rcl} 18 & = & 1T + 8 \\ +\ 25 & = & 2T + 5 \\ \hline & & 3T + 13 \end{array} \quad = 3T + (T + 3) = (3T + T) + 3 = 4T + 3 = 43$$

Form 2:

$$\begin{array}{r} 18 \\ +\ 25 \\ \hline 13 \\ +\ 30 \\ \hline 43 \end{array} \qquad \text{or} \qquad \begin{array}{r} 18 \\ +\ 25 \\ \hline 30 \\ +\ 13 \\ \hline 43 \end{array}$$

When you write Form 2 on the board, be sure that the children understand where the 13 and the 30 come from. You may want to write it on the board as follows:

$$\begin{array}{r} 18 \\ +25 \\ \hline 13 \\ +30 \\ \hline 43 \end{array} \qquad \begin{array}{l} 13 = 8 + 5 \\ 30 = 10 + 20 \end{array}$$

Be sure that the children understand that you wrote the equations on the right just for clarification and do not expect them to do it that way every time.

Ask the children whether they think it makes any difference if they add the tens column or the ones column first when they use Form 2 above. They should see that they get the same answer by either method.

There are two reasons for introducing addition from left to right and from right to left at the same time. First, it allows the children to be more flexible in working with computation. If alternative methods are given, the children are more likely

to understand the basic process rather than to memorize mechanical procedures. Second, there will be times when the children will find left-to-right addition especially useful. For example, when they learn to work with multi-digit numbers, they can get a quick estimate of a sum by adding only the numerals with corresponding place value at the extreme left. (In adding 8,330,411 and 7,138,499, a good estimate of the sum can be obtained by adding 8 million and 7 million.)

Next write the following form on the board:

Form 3:

$$\begin{array}{r} {\scriptstyle 1} \\ 18 \\ +\ 25 \\ \hline 43 \end{array}$$

Form 3 is a further condensation of Form 2, but it can only be used with adding from right to left—that is, the ones column must be added first, and then the tens column.

Worksheets 32, 33, and 34 provide practice in adding two-digit numbers with regrouping. Children may choose to use grid pieces, charts, or an abacus. The problems are presented in Form 2 notation. Help those who prefer to use the longer form (Form 1) to relate it to the condensed form used on the worksheets. For example, a child who looks at the problem $\begin{array}{r} 43 \\ +\ 28 \\ \hline \end{array}$, may prefer to use the longer form. He might write on scratch paper:

$$\begin{array}{rcl} 43 & = & 4T + 3 \\ 28 & = & 2T + 8 \\ \hline & & 6T + 11 \end{array}$$

Then he can transfer his results to Form 2, as follows:

$$\begin{array}{r} 43 \\ +\ 28 \\ \hline 60 \\ 11 \\ \hline \end{array} \qquad \text{or} \qquad \begin{array}{r} 43 \\ +\ 28 \\ \hline 11 \\ 60 \\ \hline \end{array}$$

Finally he can get his answer by adding 60 + 11 in the vertical form. This should help him make the transition to using the more condensed notation.

Worksheet 35 provides a review of estimating sums. The children may use any method of addition that they prefer.

NOTE: Have the children save their place value charts for Lesson 12.

Worksheet 32
Unit 20 Name _____

Add to find the answers.

Sample problem.

$$\begin{array}{r} 43 \\ +\ 28 \\ \hline 11 \\ 60 \\ \hline 71 \end{array}$$

$$\begin{array}{r} 48 \\ +\ 23 \\ \hline 11 \\ 60 \\ \hline 71 \end{array}$$

$$\begin{array}{r} 39 \\ +\ 12 \\ \hline 11 \\ 40 \\ \hline 51 \end{array}$$

$$\begin{array}{r} 18 \\ +\ 18 \\ \hline 20 \\ 16 \\ \hline 36 \end{array}$$

Worksheet 33
Unit 20 Name _____

Add to find the answers.

$$\begin{array}{r} 57 \\ +\ 16 \\ \hline 13 \\ 60 \\ \hline 73 \end{array}$$

$$\begin{array}{r} 32 \\ +\ 48 \\ \hline 10 \\ 70 \\ \hline 80 \end{array}$$

$$\begin{array}{r} 69 \\ +\ 29 \\ \hline 90 \\ 18 \\ \hline 98 \end{array}$$

$$\begin{array}{r} 52 \\ +\ 38 \\ \hline 80 \\ 10 \\ \hline 90 \end{array}$$

Worksheet 34
Unit 20 Name _____

Add to find the answers.

19	74
+ 15	+ 18
14	12
20	80
34	92

45	64
+ 35	+ 27
10	11
70	80
80	91

Worksheet 35
Unit 20 Name _____

Estimate your answer before you add.
Then work the problem.
Is your answer close?

Lower estimate		Upper estimate	Lower estimate		Upper estimate
60	< 67 <	70	50	< 59 <	60
+ 10	< + 12 <	+ 20	10	< + 16 <	20
70	< 79 <	90	60	< 75 <	80
70	< 72 <	80	40	< 46 <	50
10	< + 18 <	20	20	< + 23 <	30
80	< 90 <	100	60	< 69 <	80
10	< 18 <	20	20	< 22 <	30
40	< + 44 <	50	30	< + 39 <	40
50	< 62 <	70	50	< 61 <	70

Lesson 12: SUBTRACTION WITH REGROUPING

In this lesson the algorithm for subtracting with regrouping is developed. The grid pieces from Lesson 6 are used again.

MATERIALS

- flannel board and felt numerals (optional)
- masking tape

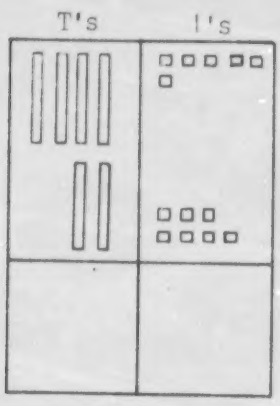
-- for each child --

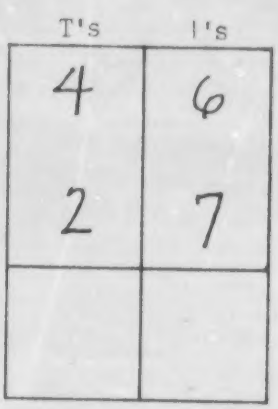
- grid pieces (from Lesson 6)
- place value chart (from Lesson 7)
- blank paper
- addition slide rule
- abacus (optional)
- Worksheets 36, 37 and 38

PROCEDURE

Put a subtraction problem and charts similar to the ones below on the chalkboard and/or flannel board. Use the 1-pieces and 10-pieces to demonstrate the problem. The children should follow the demonstration by manipulating their grid pieces on their place value charts.

46
-27





CAN WE TAKE SEVEN 1-PIECES AWAY FROM SIX 1-PIECES? (No.)

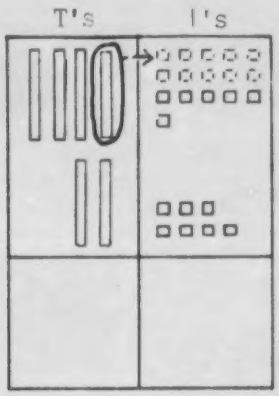
Ask how the problem might be solved. If no one suggests it, ask:

WHAT DID WE DO WHEN WE WERE ADDING AND WE HAD MORE THAN NINE PIECES IN THE ONES PLACE? (We exchanged ten 1-pieces for a 10-piece.)

CAN WE EXCHANGE A 10-PIECE FOR TEN 1-PIECES? HOW WILL OUR CHART LOOK?

Have a child make the exchange.

46
-27

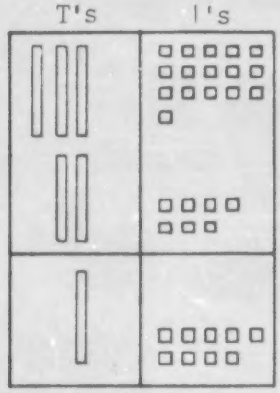


Then write out:

$$46 = 4T + 6 = 3T + 16$$

Now the chart showing numerals can be filled in, and the subtraction can be completed.

46
-27



T's	1's
3 4	16 6
2	7
1	9

Have the children do the same problem on their addition slide rules and note that they get the same answer.

Take enough time here to be sure that the children understand that numbers can be represented in many ways. Problems of the following type should be presented.

37 = 3 tens + 7 ones = ____ tens + 17 ones

25 = 2 tens + 5 ones = 1 ten + ____ ones

The first part of Worksheet 36 should be worked now if the children seem ready for it.

Worksheet 36
Unit 20

Name _____

Fill in the boxes.

49 = [4] T + [9] = 3 T + [19]

27 = [2] T + [7] = 1 T + [17]

63 = [6] T + [3] = [5] T + 13

38 = [3] T + [8] = [2] T + 18

Use your pieces and your place value chart.

	T's	1's
	6	0
-	2	4
	3	6

	T's	1's
	3	2
-	1	9
	1	3

	T's	1's
	2	2
-	1	3
	0	9

	T's	1's
	9	1
-	8	6
	0	5

	T's	1's
	2	4
-	1	3
	1	1

83
- 68
15

When the children have worked several problems using the charts, show them ways the procedure can be recorded without the charts. The following forms are suggested. Don't urge a child to use the more concise form until he is at ease with the preceding ones. It is not expected that all the children will be able to master Form 3.

Remind the children that subtracting 27 is the same as subtracting 20 and then subtracting 7. This is why we can write the subtraction problem as - 2T - 7.

Form 1:

$$\begin{array}{r} 46 \\ -\,27 \\ \hline \end{array} \quad = \quad \begin{array}{r} 4T + 6 \\ -\,2T - 7 \\ \hline \end{array} \quad = \quad \begin{array}{r} 3T + 16 \\ -\,2T - 7 \\ \hline T + 9 = 19 \end{array}$$

Form 2:

$$\begin{array}{r} 46 \\ -\,27 \\ \hline \end{array} \quad = \quad \begin{array}{r} 40 + 6 \\ -\,20 - 7 \\ \hline \end{array} \quad = \quad \begin{array}{r} 30 + 16 \\ -\,20 - 7 \\ \hline 10 + 9 = 19 \end{array}$$

Form 3:

$$\begin{array}{r} {}^{3}\,{}^{16} \\ \not{4}\not{6} \\ -\,27 \\ \hline 19 \end{array} \qquad \text{and} \qquad \begin{array}{r} 46 \\ -\,27 \\ \hline 19 \end{array}$$

The children should now finish Worksheet 36 and do Worksheets 37 and 38. Worksheet 38 reviews the inverse relation of addition and subtraction. Most children will need extra paper to complete the problems and some may wish to use an abacus, place value chart and grid pieces, or Minnebars.

Worksheet 37
Unit 20 Name _____

Finish these problems.

Sample

$42 - 18 = 4T + 2 - T - 8 = 3T + 12 - T - 8$

$2T + 4 = 24$

$94 - 39 = 9T + 4 - 3T - 9 = 8T + \boxed{14} - 3T - \boxed{9}$

$5T + 5 = 55$

$61 - 47 = 6T + 1 - 4T - 7 = \boxed{5}T + \boxed{11} - 4T - \boxed{7}$

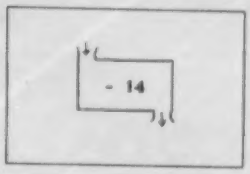
$1T + 4 = 14$

$50 - 38 = 5T + 0 - 3T - 8 = \boxed{4}T + \boxed{10} - \boxed{3}T - \boxed{8}$

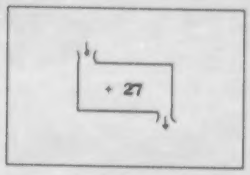
$1T + 2 = 12$

Worksheet 38
Unit 20 Name _____

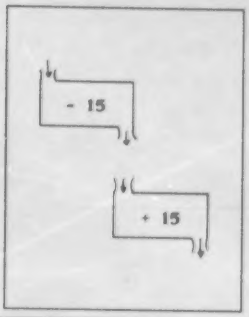
Put each number through the machines: 36, 23, 72, 18



In	Out
36	22
23	9
72	58
18	4



In	Out
36	63
23	50
72	99
18	45



In	Middle	Out
36	21	36
23	8	23
72	57	72
18	3	18

Lesson 13: POWERS OF TEN; ADDING THREE-DIGIT NUMBERS

The children are introduced to the notation of T^0, T^1, and T^2 for one, ten, and one hundred. Then they add three-digit numbers using the procedure developed in Lesson 11.

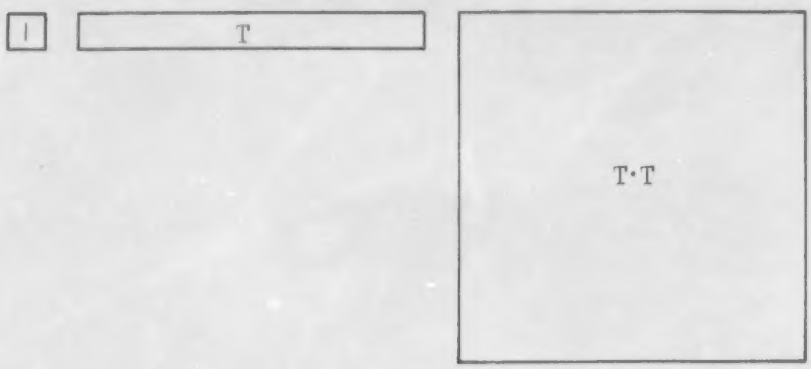
MATERIALS

- grid pieces (from Lesson 6)
- abacus
- labels or tape for abacus
- Worksheets 39 through 42

PROCEDURE

Activity A

On the chalkboard draw diagrams of the children's grid pieces.



Write the following table on the board and have the children fill it in.

____ ones = 1 ten	____ · 1 = 1 T
____ tens = 1 hundred	____ · T = 1 T · T

Explain to the class that because it takes T (or ten) 1-pieces to make a 10-piece and it takes T (or ten) 10-pieces to make a 100-piece, we say we are "grouping by tens."

During the following discussion, label the diagrams that you drew earlier.

WHEN WE GROUP TEN 1-PIECES WE GET A 10-PIECE.

WE CAN CALL THIS PIECE T^1. THE T TELLS US THAT WE ARE GROUPING BY TENS, AND THE 1 TELLS US THAT WE HAVE GROUPED BY TEN JUST ONCE.

NOW SUPPOSE WE GROUP TEN T^1-PIECES. (Ten tens.)

WHAT CAN WE CALL THAT? (T^2.)

THE T TELLS US THAT WE ARE GROUPING BY TENS, AND THE 2 TELLS US THAT THE PROCESS OF GROUPING BY TENS HAS BEEN DONE TWICE.

WHEN WE ARE GROUPING THIS WAY, WHAT CAN WE CALL A 1-PIECE? LET US WRITE DOWN A T TO TELL US THAT WE ARE GROUPING BY TENS.

NOW, HOW MANY TIMES HAVE WE GROUPED BY TENS? (None.)

WHAT CAN WE WRITE TO TELL US THAT? (T^0.)

Each child should write T^0, T^1 and T^2 on all the pieces in his set. Call attention to the fact that T^2, $T \cdot T$, and 100 are all names for the same number.

By the same procedure, other powers of T such as T^3, T^4, T^5, etc. can be explained. Do this only if the class expresses interest.

Explain to the children how to read T notation:

T^0 is "T to the zero power."

T^1 is "T to the first power."

T^2 is "T to the second power," etc.

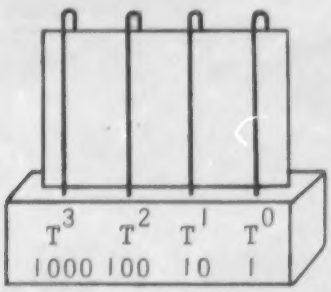
However, we sometimes say it a shorter way:

T^0 is "T to the zero."
T^1 is "T to the one."
T^2 is "T to the two," etc.

When the children study the T^2 diagram they should find it easy to understand why T^2 is commonly called "T squared."

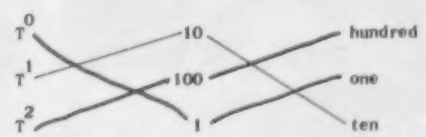
Activity B

Label an abacus with the powers T^0, T^1, T^2, and higher if you desire. Also label the places 1, 10, 100, etc.



Worksheet 39
Unit 20 Name _____

Connect all the names for the same numbers.



Finish these problems.

100 = [10] tens
200 = [20] tens
300 = [30] tens

30 = [3] T^1
40 = [4] T^1
60 = [6] T^1

365 = 300 + 60 + 5 = $3T^2$ + $6T^1$ + [5] T^0
782 = 700 + [80] + [2] = [7] T^2 + [8] T^1 + [2] T^0
912 = [900] + 10 + [2] = [9] T^2 + [1] T^1 + [2] T^0
695 = 600 + [90] + [5] = [6] T^2 + [9] T^1 + [5] T^0

Show a number on the abacus and have the children write on the chalkboard the numeral in the following form.

$345 = 300 + 40 + 5$

$= 3T^2 + 4T^1 + 5T^0$

Repeat this with different numerals until you feel that the children are ready to complete Worksheet 39.

Activity C

Put the following problem on the board and have the children guide you in solving it. Encourage them to use their T^0, T^1 and T^2 pieces if they wish.

	T^2 100's	T^1 10's	T^0 1's
324	3	2	4
+538	5	3	8

Also record it this way:

$$\begin{array}{rcrcrcl} 324 & = & 300 & + & 20 & + & 4 \\ +538 & = & +\ 500 & + & 30 & + & 8 \\ \hline & & 800 & + & 50 & + & 12 \end{array} \qquad \begin{array}{rcrcrcl} & = & 3T^2 & + & 2T^1 & + & 4T^0 \\ & = & +\ 5T^2 & + & 3T^1 & + & 8T^0 \\ \hline & = & 8T^2 & + & 5T^1 & + & 12T^0 \end{array}$$

Ask the children if they should have so many ones in the ones place. Have them guide you through the regrouping, and under the previous answers, write the following:

$$800 + 60 + 2 = 862 = 8T^2 + 6T^1 + 2T^0$$

Some children will be ready to record their problems in the shorter forms:

$$\begin{array}{r} 324 \\ +538 \\ \hline 12 \\ 50 \\ 800 \\ \hline 862 \end{array} \qquad \begin{array}{r} 324 \\ +538 \\ \hline 800 \\ 50 \\ 12 \\ \hline 862 \end{array} \qquad \begin{array}{r} {\scriptstyle 1}\ \ \\ 324 \\ +538 \\ \hline 862 \end{array}$$

Worksheet 40
Unit 20 Name _____

Find the sums.

321 + 143 = 464	347 + 428 = 775
292 + 132 = 424	162 + 71 = 233
349 + 217 = 566	542 + 419 = 961

Give as much group practice as necessary with three-digit addition. Be sure to include regrouping in the ones and tens place. Then have the children do Worksheet 40.

Discuss three-digit subtraction problems in a similar way. Be sure that you do enough demonstration problems that the class has no difficulty following the procedure, though not all the children may be able to do it independently. Then have the children fill out Worksheet 41.

Have the children do Worksheet 42, which gives practice in the addition and subtraction of three-digit numbers. They may use any desired form to record the operations.

Worksheet 41
Unit 20 Name _____

Find the differences.

647 - 226 = 421	224 - 193 = 31
283 - 81 = 272	576 - 227 = 349
643 - 219 = 424	827 - 136 = 291

Worksheet 42
Unit 20 Name _____

1. The school library has 532 books. 218 books were taken out. How many books are left?

532 − 218 = 314

2. A squirrel hid 58 acorns in one tree. He hid 61 acorns in another tree. How many acorns did he hide in both trees?

58 + 61 = 119

3. Tom picked 222 blueberries. Fred picked 228 blueberries. How many did the two boys pick?

222 + 228 = 450

SECTION 3 A WEATHER STATION

PURPOSE

- To provide a meaningful context for the practice of addition and subtraction.
- To continue developing the techniques of measurement, in this case working with temperature, wind speed and humidity.
- To present methods of collecting and organizing weather data.
- To make the children aware of how temperature, wind speed and humidity vary over a period of time.

COMMENTARY

In this section the children learn to read a thermometer, gauge the speed of wind and determine relative humidity. They use simple instruments to make these measurements.

The information gathered by the children and that supplied by the Student Manual is used to practice addition and subtraction. The develcpment of these computational skills is the main objective of this unit. The weather station serves as an interesting device for improving the child's skill in working with larger numbers. The children will keep daily weather records of their observations. They should keep these records for at least two weeks and may wish to continue recording for a longer period of time.

Lesson 14 provides an introduction to weather terminology by having the children classify descriptive words about the weather into sets. Lessons 15 and 16 focus on the use of the thermometer. In Lesson 17 the children are taught ways of measuring the speed of the wind. First, they use their own observations of smoke drift, flag extension, tree movement, etc. to make rough approximations of the wind speed. The approximations are refined when they learn to use an anemometer (an-uh-MOM-uh-ter).

In Lesson 18, the children learn about moisture in the air. In

Lesson 19, they are taught to use a simple hygrometer to measure the amount of moisture in the air.

In this section, little attempt is made to explain why and how the weather instruments work. Such explanations will be sought by the children in later grades.

Lesson 14: DESCRIBING WEATHER

The children compile a list of words they can use to describe the weather. These words are then classified into three sets: words dealing with air movement, with temperature and with water (moisture in the air). The purpose of this lesson is to direct the children's attention to a variety of weather conditions that they will later learn to measure.

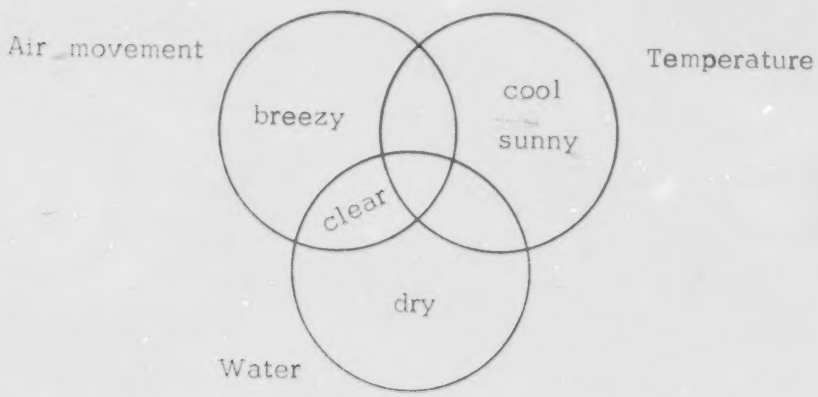
PROCEDURE

Tell the children that in the next lessons they are going to study the weather. They will build a weather station that will help them observe changes in the weather. They are going to learn how to use the instruments in their weather station and how to record weather information.

Tell the children to pretend that they are going to write a letter to a friend who lives far away. In this letter they want to tell their friend what kind of weather we are having today. Have them stand at an open window and think of words to describe the weather. As suggestions are made, list the words on the chalkboard. A typical list might include such words as cool, sunny, breezy, clear, dry.

Tell the children that you are going to make a diagram of three sets on the chalkboard. One set will contain words that tell about air movement, the second set will have words that tell about water in the air and the last set will have words that deal with temperature.

Draw three intersecting circles on the chalkboard:



Let the children decide in which area of the diagram the words already on the board belong. Write the words in the diagram according to their directions.

Then ask the children if they know of any other weather words that might be included in the diagram. Below are some of the words the children might suggest. (Obviously, no class will think of all the possible words that can be used to describe the weather.)

clear
calm
chinook
windy
wind-chill
sunny
breezy
heat wave
frost
freezing
hot
cold wave
thunder
dew
drought
precipitation
squall
rainbow
hurricane
tornado
blizzard
thunderstorm
ice
snow
sleet
discomfort index
humidity
rain
cloudy
dry
fog
mild

Allow the child who makes a suggestion to indicate where in the diagram that word should go. You may wish to suggest some words and ask the children to tell you where they should be placed on the diagram. Help the children spell the words if necessary. Do not be too concerned with accuracy of placement, especially since the classification of many words may be debatable. The main purpose of this activity is to call attention to the great variety of weather conditions and to the interaction of different properties in determining them.

Lesson 15: READING THE THERMOMETER

In this lesson, the children learn to read the thermometer and to record their readings.

MATERIALS

- 1 kit thermometer
- transparency of thermometer (printed original included on page 117)
- overhead projector
- large demonstration thermometer (optional)
- red crayons, 1 per child
- Worksheets 43 through 48

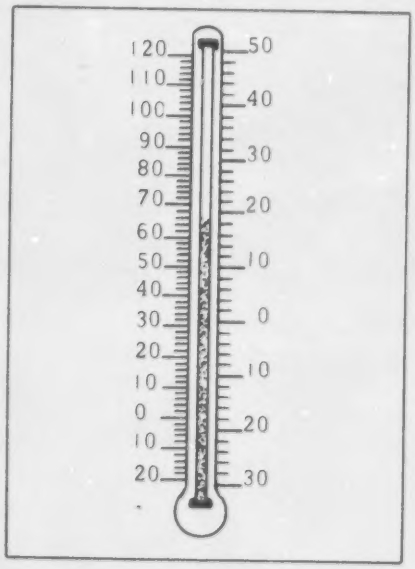
PREPARATION

If you do not have an overhead projector, you could draw a large thermometer on the chalkboard, or you may prefer to make a large demonstration thermometer. The instructions for assembling one are given below.

1. Copy the transparency thermometer on a large piece of tagboard. Cut slits about one-fourth inch by three-fourths inch at the top and bottom of the indicator column on the tagboard.

2. Cut a piece of twill tape to a length twice the distance between the two slits, plus one inch. Color one-half the length of twill tape red.

3. Run the twill tape through the slits and join the ends together firmly. The loop of tape will slide so that you can set the thermometer at different temperature readings.



PROCEDURE

Tell the children that they are going to learn how to use an instrument that the weatherman uses to measure how warm or cold the weather is. Hold up a thermometer and ask if anyone knows what it is called. (Most of the children will probably know that it is called a thermometer.)

Exhibit the large demonstration thermometer, show the transparency with the overhead projector, or point out the thermometer you have drawn on the chalkboard. The scales on this thermometer are the same as the scales on the actual thermometers that the children will be working with later. To the left of the indicator column is a Fahrenheit scale. There are lines every 2 degrees and numerals at 10-degree intervals. To the right of the indicator column is a Celsius (centigrade) scale, marked off with lines at 2-degree intervals and with numerals at 10-degree intervals.

Use the Fahrenheit scale, but explain briefly to the children that the Celsius (centigrade) scale is used by scientists and by people of other countries. If the children are confused by the presence of two scales, cover the Celsius scale with a piece of masking tape.

Tell the children that you are going to set the demonstration thermometer at a certain reading. Set it at 30°. (To use the transparency, cut it along the dotted line and lay the half with the darkened indicator column printed on it over the half with the thermometer on it. To change the temperature readings, move the indicator column up or down.) Ask if anyone can tell the temperature indicated on the thermometer. You may have to explain that to read a thermometer, one should look at the top of the indicator column to see what mark on the number line is next to it.

Repeat this procedure, choosing a succession of increasingly more difficult readings, for example, 40°, 20°, 22°, 4°, 0°, 35°, 3°, 25°, 39°. Tell the children that in some places the thermometer reading goes below the zero mark in winter. Illustrate this by placing the demonstration model at 5° below zero.

WHEN THE THERMOMETER LOOKS LIKE THIS WE SAY THAT THE TEMPERATURE IS 5 DEGREES BELOW ZERO. SOMETIMES WE JUST SAY "5 BELOW."

Show other below-zero settings and ask various students to read the temperature. Be sure that the children's responses include the words "below zero."

Tell the children that now you are going to give them a few more temperature settings and this time you are going to write their answers on the chalkboard. If, for example, the setting you give is 20° and a child gives the correct response, print the following on the chalkboard:

Twenty degrees

Then ask the children:

WHAT IS A SHORTER WAY OF WRITING TWENTY? (20.)

Place the numeral under the word.

MANY PEOPLE WRITE THE WORD "DEGREES" A SHORTER WAY.

Place the symbol $^{\circ}$ after the numeral. Now you will have this on the chalkboard:

Twenty degrees

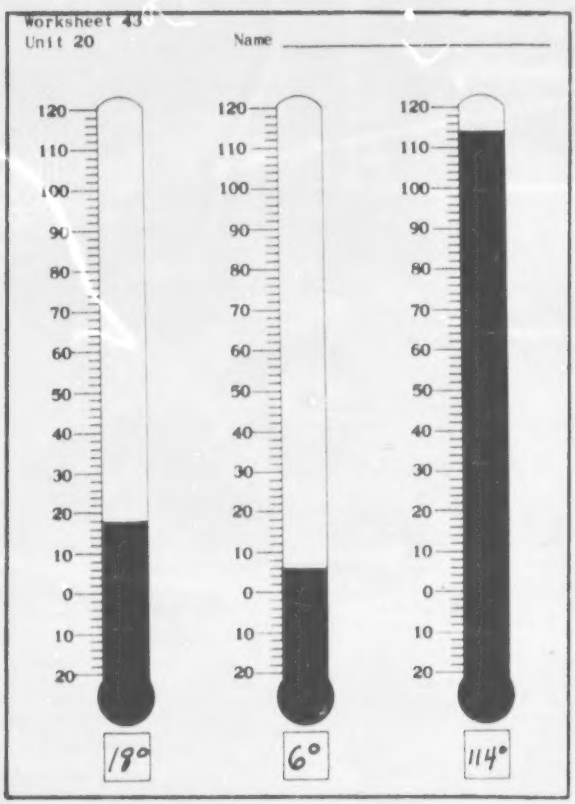
20°

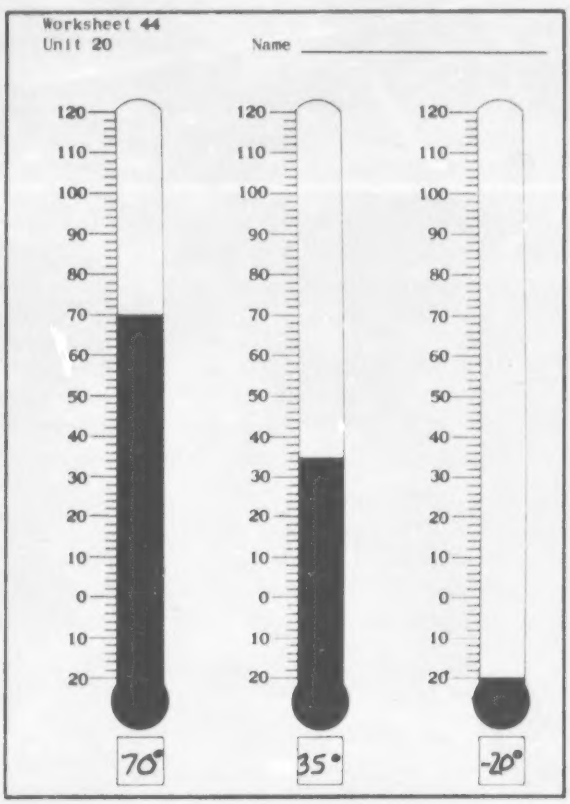
Repeat this procedure using other above-zero readings.

After most of the children have mastered the symbolism, set the demonstration thermometer scale at 10° below zero. Tell the children to think of the scale on the thermometer as a number line. Turn the demonstration thermometer sideways. Draw a number line on the chalkboard. Number several units to the right of zero and then ask if someone can tell you how to write the numbers to the left of zero. When the children's memory of negative notation is refreshed, have a child write

the demonstration thermometer reading on the board (-10). Change the setting and have another child record it. Then return the thermometer to its vertical position and provide several more examples of below-zero settings for the children to record. (Be sure that they understand that it makes no difference when you change the position of a number line or a thermometer from horizontal to vertical.)

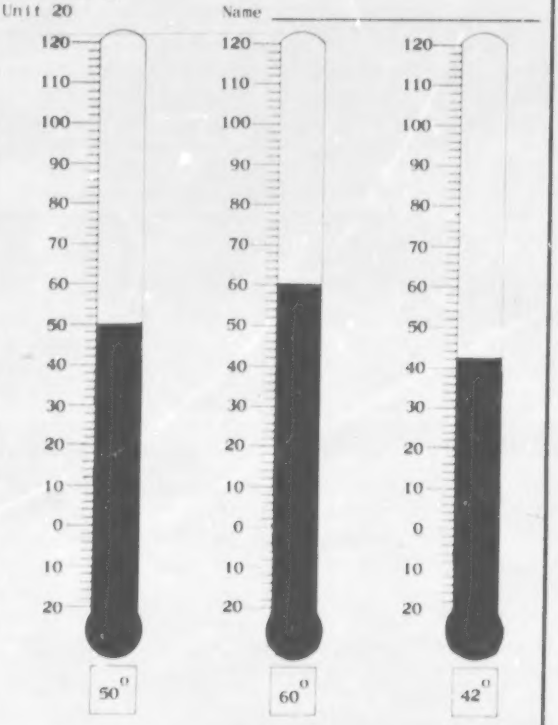
Have the children turn to Worksheet 43. They will see that on each thermometer the columns have been drawn in. They are to record the temperature readings in the empty boxes below each thermometer. This activity continues on Worksheet 44. On Worksheets 45 through 48, the children will find the temperature written in the boxes below the thermometers. They are to use red crayons to draw the columns of the thermometers to correspond to the readings.





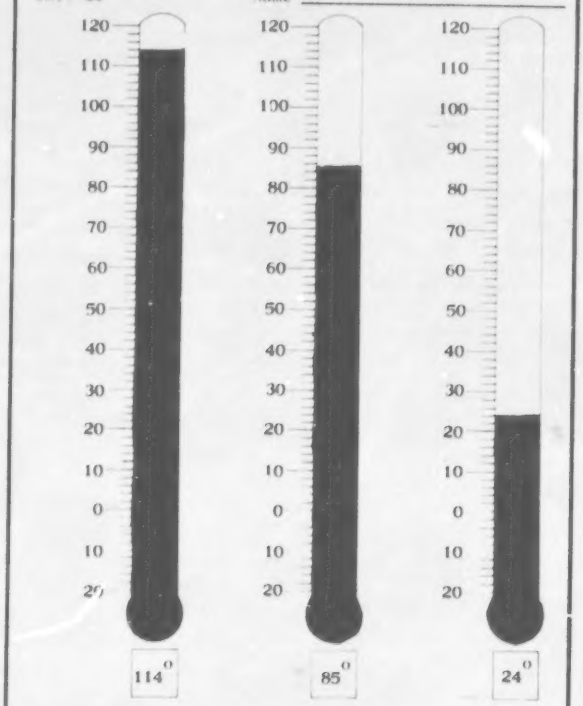
Worksheet 45
Unit 20

Name _____



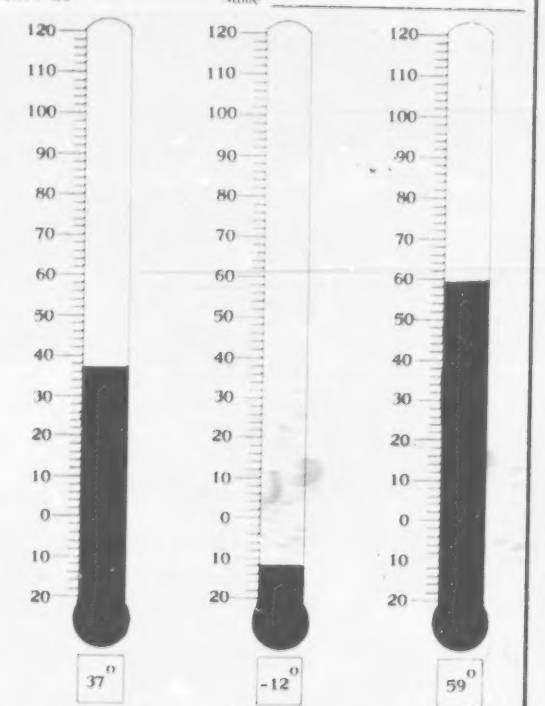
Worksheet 46
Unit 20

Name _____



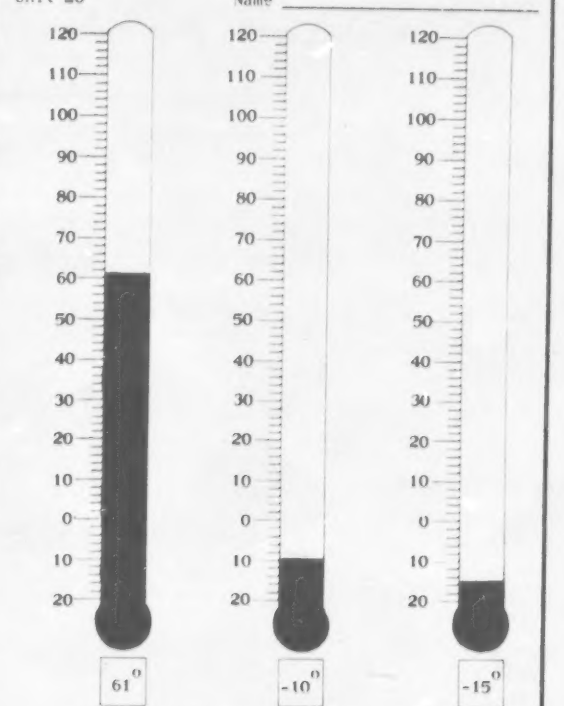
Worksheet 47
Unit 20

Name _____



Worksheet 48
Unit 20

Name _____



Lesson 16: OBSERVING TEMPERATURE CHANGES

In this lesson, the children make actual thermometer readings and record temperatures on tables and graphs. They study temperature variations throughout a day and begin recording data on their Daily Weather Records. This lesson will probably take 4 days.

MATERIALS

- 14 thermometers
- red crayons, 1 per child
- masking tape
- Worksheets 49 through 59

PREPARATION

Thermometers should be placed outdoors on the north, east, south and west walls of the school building. Use masking tape. If you cannot find any place to which the tape will stick, tape each thermometer to the top of a large carton. Before you conduct Activity B, have four reliable children place these boxes outdoors, alongside the appropriate wall. Show them which is the north (etc.) wall the first time so they can find it by themselves next time. Tape a fifth thermometer in a corridor at about the average eye level of your class.

PROCEDURE

Activity A

Give a thermometer to each group of three or four children. Ask each child to write on scratch paper the temperature reading shown on his group's thermometer. The readings made by different groups may vary one or two degrees because of differences in the thermometers due to manufacturing methods. Show the children how to hold the thermometer for reading so that the top of the red column is at eye level and directly in front of them. Have them try holding the thermometer higher and lower than eye level and to one side, in order to see the distortions that can occur. Also,

instruct the children not to hold the thermometer by the bulb at the bottom. To illustrate your point have the children place their thumbs lightly on the bulb and then observe what happens to the liquid column.

Activity B

Now have the children read the temperature on each of the thermometers that were placed indoors and outdoors around the building. To avoid confusion, divide the class into groups and send them out at intervals. The children should take Worksheet 49 with them and record the readings on it. It may be necessary to remind the children which is the north side of the building, which is the south side, etc. After everyone has had an opportunity to make his observations, conduct a short discussion about the differences among the temperatures they have recorded. Some factors influencing temperature differences are whether the thermometers are indoors or outdoors, in the shade or in the sun.

Worksheet 49
Unit 20 Name _____

Read the temperature on each of the thermometers.
Record the temperatures here.

North

East

South

West

Hall

Have the thermometers brought back into the classroom. Set aside an area of the room where the weather instruments can be displayed when they are not being used. This will be the weather station. Perhaps one of the children would like to make a sign for the weather station.

Tell the children that the next day outdoor readings will be taken every hour and they will record their observations on graphs and tables. Say that later they will learn to use other instruments to make other weather observations.

Activity C

The next morning have a thermometer put on each of the four sides of the school building as you did for Activity B.

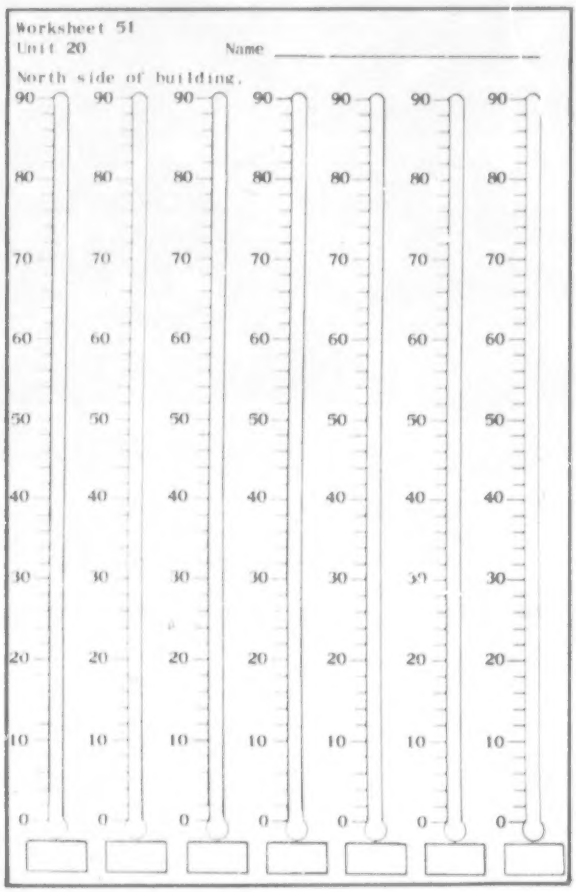
Worksheet 50
Unit 20 Name _____

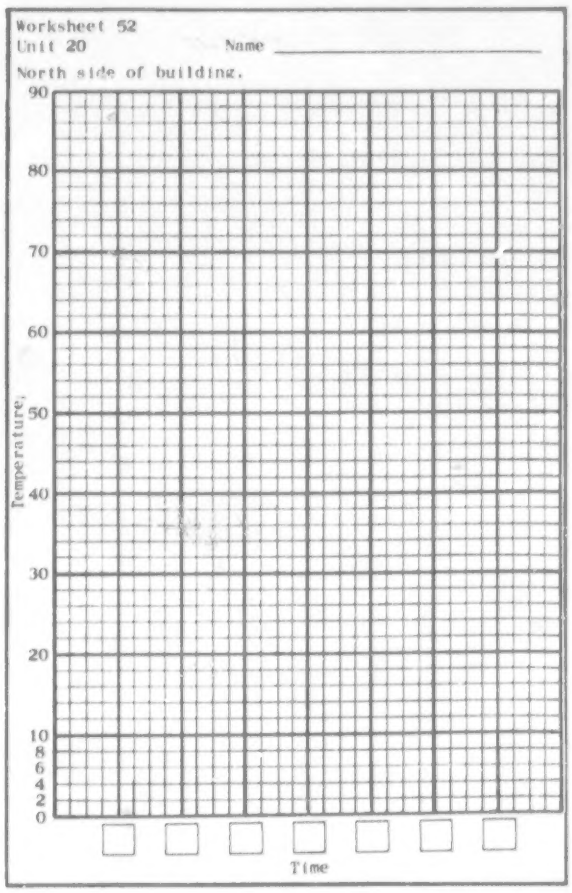
Record the temperatures.

Date _____

Time	North	East	South	West

Select two reliable children and each hour throughout the day have them read each of the thermometers and record their data on Worksheet 50 which has space for seven temperature readings. The day after this information has been collected and recorded, put it on the chalkboard and have the other children record it on their copies of Worksheet 50. Then have them carefully tear out Worksheet 50 and turn to Worksheets 51 and 52. They should place Worksheet 50 alongside Worksheet 51, because they will be using the data in column 2 on Worksheet 50 to fill out 51 and 52. Worksheet 51 contains drawings of seven thermometers, on





which the children are to record with a red crayon the hourly temperatures that were read on the previous day at the north side of the building. In the boxes along the horizontal axis, they should write the time at which the readings were made. When they have finished coloring the thermometer columns, they are to graph the hourly temperatures for the north side of the building on Worksheet 52. Again, the children should write in the boxes along the horizontal axis the time at which the readings were made. (Some children may be able to graph directly from the data in column 2 on Worksheet 50. Only those students who need the transitional step provided by Worksheet 51 should fill it out.) You may need to review what the children learned about graphing in Unit 19, Change and Function. Tell them that the ordered pair consists of the hour (the first number) and the temperature (the second number); e.g., (10 a.m., 44°). They are to go to 10 a.m. on the "over" axis and from 0° to 44° on the "up" axis; where the grid lines intersect, they make a red dot. (There are grid lines every two degrees.) The children should use a ruler and a red crayon to draw a line from one dot to the next one to its right. Worksheets 53, 54 and 55, which are graphs for the temperature on the east, south and west sides of the building, are just like Worksheet 52 and should be filled out at this time. The children will have to refer to Worksheet 50 for the data to be graphed on these worksheets.

Ask the children which part of the day was the warmest. Follow up their replies by asking how they can be sure. Ask if they can tell by looking at Worksheets 50, 51, 52, 53, 54 or 55. (The worksheets carry no clearcut answer, and the children may make a variety of observations.)

After you have allowed adequate time for discussion, have the children circle the highest temperatures reached on each side of the building as recorded on Worksheet 50. These will probably be found at a different time of day for each side. Ask the children how they account for the differences. If no child mentions that these highs relate to the position of the sun, call attention to that fact.

Activity D

Have the children look at Worksheet 50, where they have

already circled the highest temperature of the day for each side of the building. Now instruct them to circle the lowest temperature for each side of the building. Have them subtract the lowest from the highest temperature at each side of the building to see how much variation there is. They should also look at their graphs to see the difference between a graph showing a wide range and one showing a narrow range. They should note that if one of the thermometers is placed so that the sun never shines on it, the temperature range will be much smaller than for a thermometer that is first in shade, then in sunlight.

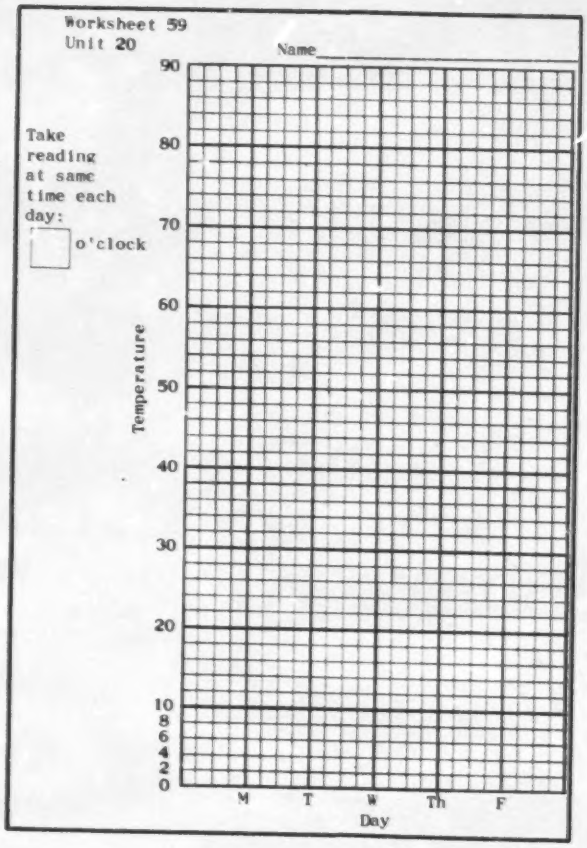
Ask the children which of the outdoor locations would be best for making daily temperature readings. They should consider such factors as least variation in temperature and shadiest side of the building. Explain that when the weatherman takes the temperature, he takes it in the shade. One reason he does it this way is that he gets a more accurate recording of the temperature of the air; the thermometer is not so much affected by the direct sunlight as by the air around it.

Worksheet 56
Unit 20

Name _____

Daily Weather Record

Date	Temperature	Wind Speed	Humidity



After the children have decided on the best place, have a different pair make temperature readings there at the same time each day and record the data on their Daily Weather Records (Worksheet 56). After they come back to the classroom, the rest of the class can copy the data onto Worksheet 56. Worksheets 56, 57 and 58 are identical except for the worksheet number. As the children work along in this section, they will learn to make the other readings that are called for. You may wish to make a large copy of Worksheet 56 on the chalkboard and have the children fill it in each day, too.

Activity E

Worksheet 59 is a graph on which the children are to record the daily temperature readings for one week. They can get the data for filling in this graph from their Daily Weather Records.

NOTE: Have the children save Worksheets 56, 57 and 58 for use in later lessons.

Lesson 17: WIND SPEED

In this lesson, the children will first make a subjective judgment of wind speed using a picture scale. Then they will use the more refined technique of measuring wind speed with a cup anemometer which consists of four cups fastened to arms that turn. The cups are caught by the wind, causing the anemometer to spin. The greater the speed of the wind, the faster the anemometer spins.

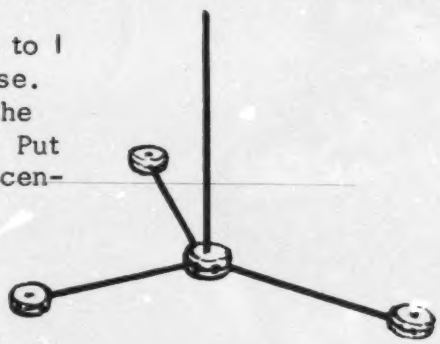
MATERIALS

- anemometer (directions for assembling are given below)
- red magic marker or crayon
- cellophane tape
- watch or clock with second hand
- transparency of wind-speed graph (printed original included on page 119)
- whistle (optional)
- Worksheets 60, 61 and 62

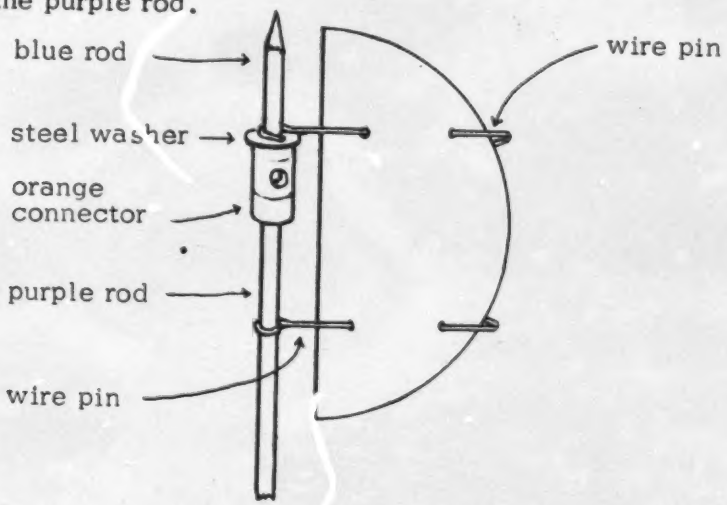
PREPARATION

Before conducting Activity B, build the anemometer. Following are detailed instructions for constructing it from the materials provided in the kit. The materials are: Tinkertoy rods, 3 green (about 7") 1 purple (about 11"), 4 yellow (about 2"), 1 blue (about 3"); 5 round Tinkertoy connectors; 1 orange cylindrical Tinkertoy connector; 1 cardboard wind vane; 2 paperclip pins; 1 steel washer; 4 plastic straws; 4 Ping-pong ball halves; 1 "Bik" pen cap.

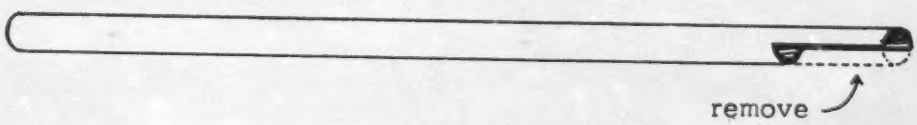
1. Connect the 3 green rods to 1 round connector to form a base. Attach a round connector to the other end of each green rod. Put the purple rod upright in the center connector.



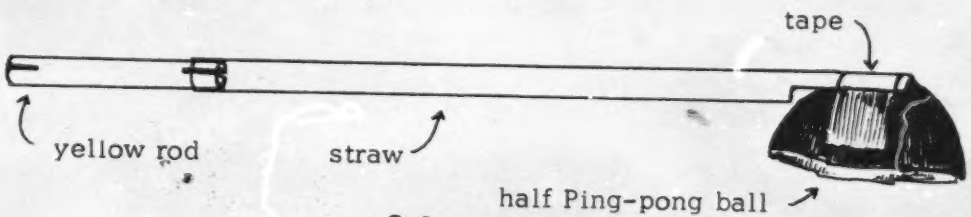
2. On the cardboard wind vane, make four holes as shown in the illustration. Use a sharpened pencil point. Insert the two wire pins in the holes and bend back the ends of the pins. Make a point on the blue rod by using a pencil sharpener; then insert it in the orange cylindrical connector. Slip the steel washer over the blue sharpened rod. Slide the top wire pin of the wind vane over the blue sharpened rod, on top of the washer. Slide the bottom vane pin over the purple vertical rod and place the orange cylindrical connector over the purple rod.



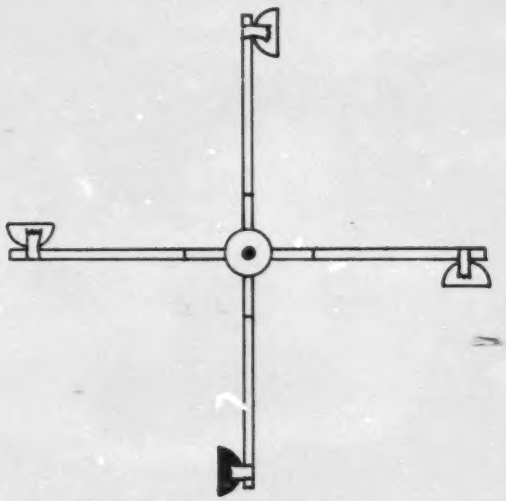
3. Take a plastic straw and at one end cut down the length for one inch and remove half of the cut part. Do this with the other three straws.



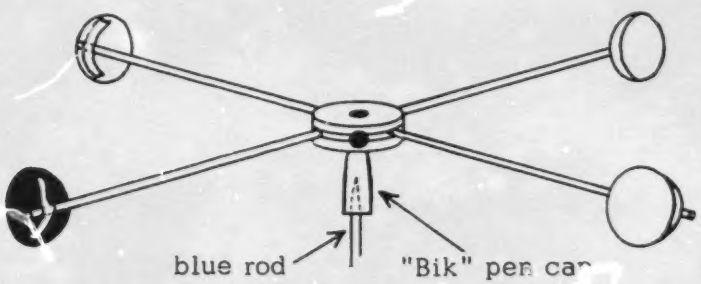
4. Slide the uncut end of the plastic straw over one end of a yellow rod. Attach half a Ping-pong ball to the cut end of the straw, using cellophane tape. Be sure to center the straw on the half Ping-pong ball. Repeat this procedure with the other three plastic straws and balls. Use a magic marker or a crayon to color one of the Ping-pong balls red.



5. Take a round connector and attach the four arms as shown below. Make sure that all the Ping-pong ball cups are facing as shown.



6. Push the "Bik" pen cap into the center of the round connector that bears the four arms. Place this whole assembly over the sharpened blue rod so that the open end of the pen cap is over the top of the point.



PROCEDURE

Activity A

Children are well aware that winds vary a great deal in speed and direction. All of them have tried to fly a kite or have walked against a stiff wind, or have seen the damage caused by a wind storm.

Ask the children how they can tell when wind is moving fast. Answers might include feeling the wind on the face, seeing the flag standing out, seeing papers or leaves flying, etc.

Tell the children about Sir Francis Beaufort. He was an admiral in the British Navy in the early 1800's. At that time, there were no instruments to measure exactly how fast the wind was blowing. It was especially important to know the speed of the wind because all of the larger ships used sails. By knowing how fast the wind was blowing, the captain of the ship would know how many sails to use. Sir Francis Beaufort worked out a way to determine wind speed. He selected the set of counting numbers from 0 through 12 to stand for different speeds. The scale began at 0, which was a calm. The number 2 stood for a wind he defined as "that in which a well-conditioned man-of-war, with all sail set, and clean full, would go in smooth water from 1 to 2 knots." (A knot is a speed of about 1 mile per hour.) Number 12 stood for a wind "that which no canvas could withstand."

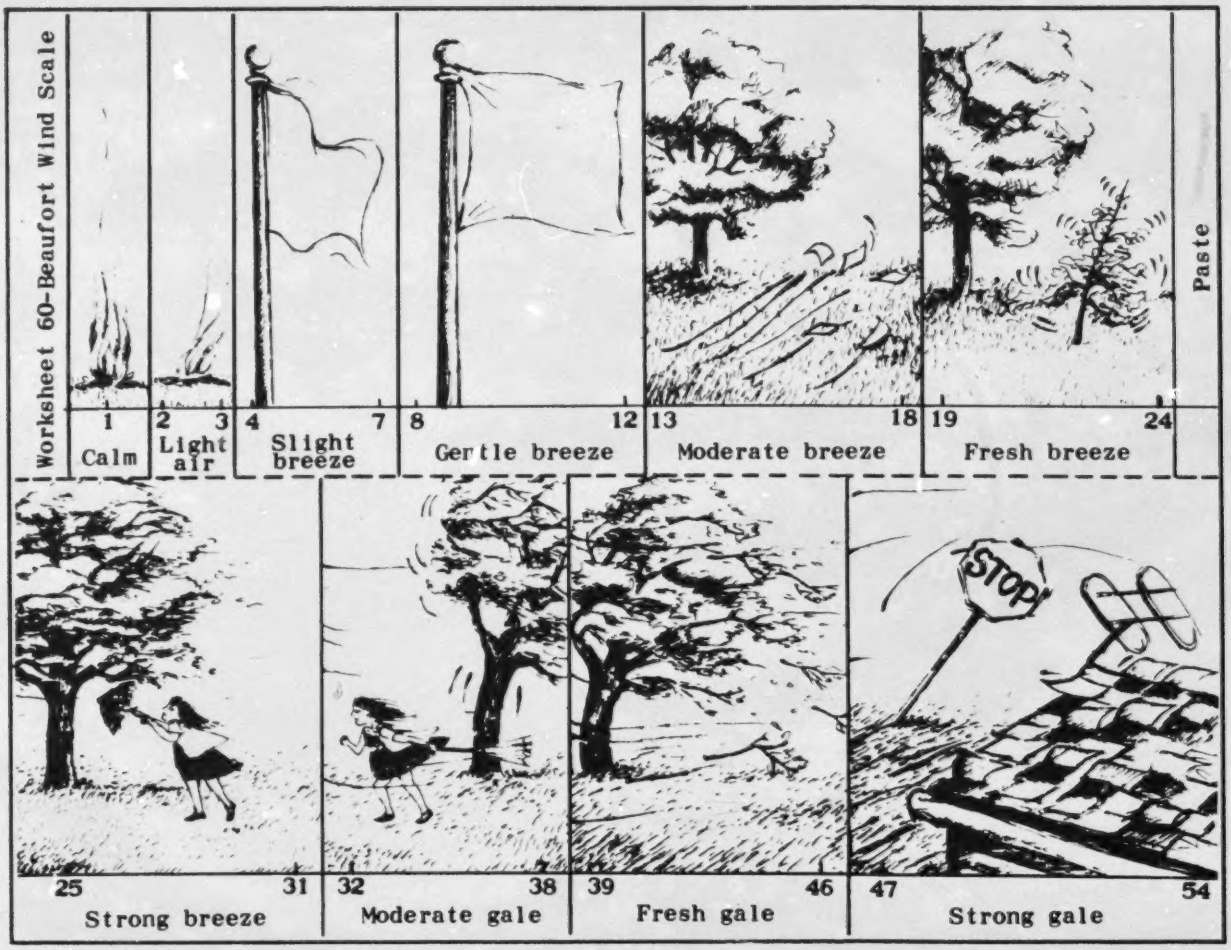
The complete scale is given on page 92 in modified terms that correspond to the children's version of the Beaufort Scale (Worksheet 60). An extra copy of this page is included in the printed materials for this unit. You might post it on the bulletin board if there are some children who are interested; however, this complete scale is not to be emphasized strongly.

On page 93 is a picture of a man-of-war under full sail, in a wind measuring 2 on the Beaufort Wind Scale. The same picture faces Worksheet 60 in the Student Manual.

Beaufort Scale for Judging Wind Speed

Land	Sea	Wind	Speed in miles per hour	Beaufort Number
Smoke rises straight up.	Mirror-smooth.	CALM	less than 1	0
Direction shown by smoke drift, but not by wind vanes.	Small wavelets like scales, but no foam crests.	LIGHT AIR	1-3	1
You feel the wind on your face, leaves rustle, wind vanes move.	Waves are short and more pronounced.	SLIGHT BREEZE	4-7	2
The wind extends a little flag and keeps leaves and small twigs in motion.	Crests begin to break; foam has glassy appearance, not yet white.	GENTLE BREEZE	8-12	3
Wind raises dust and loose paper and keeps small branches in motion.	Waves are longer; many white caps.	MODERATE BREEZE	13-18	4
Wind sways small trees in leaf.	White foaming crests everywhere.	FRESH BREEZE	19-24	5
Large branches begin to move, telephone wires whistle.	Larger waves form; foaming crests more extensive.	STRONG BREEZE	25-31	6
Whole trees sway; it is hard to walk against wind.	Sea heaps up; foam begins to blow in streaks.	MODERATE GALE	32-38	7
Twigs break off the trees.	Foam blown in dense streaks.	FRESH GALE	39-46	8
Roofs are damaged.	Waves increase visibly.	STRONG GALE	47-54	9
Whole trees are uprooted.	High waves with long overhanging crests.	WHOLE GALE	55-63	10
Widespread damage, as around the edges of hurricanes and tornadoes.	Waves so high that ships are hidden in the troughs.	STORM	64-75	11
Devastation.	Devastation.	HURRICANE	75 and up	12

92



A Beaufort Scale adapted to a child's vocabulary is provided in the Student Manual. Have the children remove Worksheet 60, the Wind Scale. If they cut along the dotted lines and paste the ends together as indicated, they will have a long picture scale that provides a guide for judging wind speed. Ask the children how they can tell if the wind is blowing. (They can feel it; they can see things being moved by it.) Have them use their scale to describe the strength of the wind. Repeat this activity several times during the day.

Activity B

The Beaufort Scale for judging wind speed is at best a crude method of approximation. Tell the class that a much more precise reading of wind speed can be obtained by using an instrument that the weatherman uses called an anemometer (AN-uh-MOM-uh-ter). Show the children the anemometer that you have assembled, and allow a few minutes for them to play with it. Then ask if they can guess how it works. (It spins in the wind.) Have them try blowing it to make it turn at different speeds.

HOW DID IT TURN WHEN YOU BLEW HARD? (Fast.)

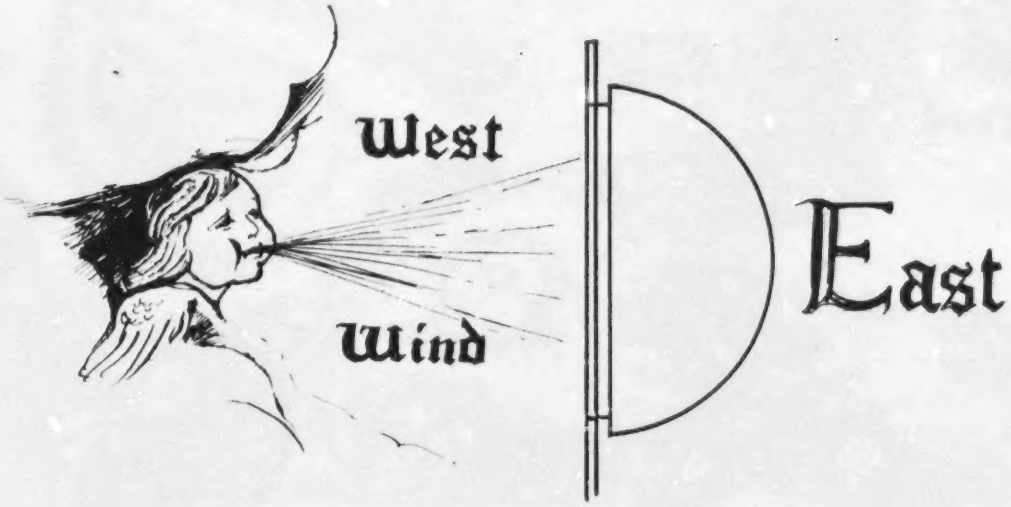
HOW DID IT TURN WHEN YOU BLEW GENTLY? (Slowly.)

HOW CAN THE ANEMOMETER HELP US FIND THE WIND SPEED? (We can tell the speed by how fast it turns.)

Explain that the speed of the wind will be determined by how many times the red cup of the anemometer goes around in a given period of time. The children's job is to count each time the red cup comes around to a certain spot.

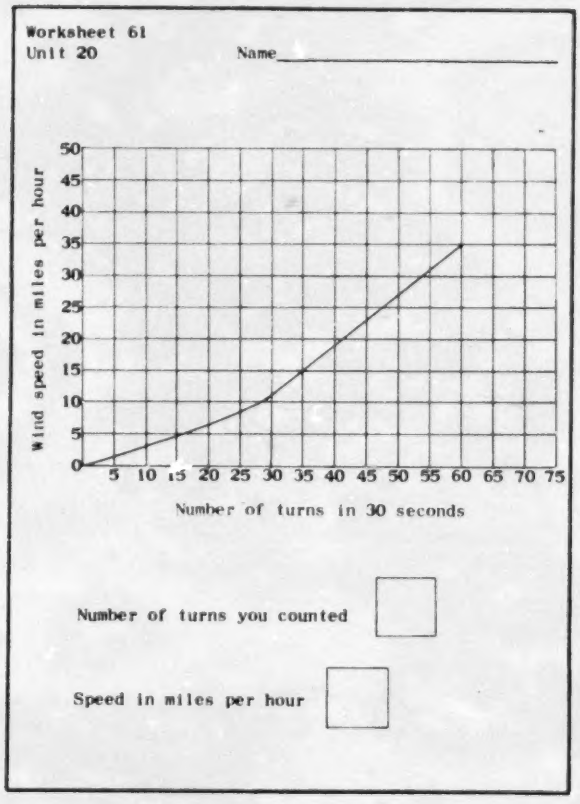
Take the children and the anemometer outside on the playground or to some other unsheltered area. Bring along a watch with a second hand. Instruct the children to begin counting when you give the signal and to stop counting when you give the signal again. (A whistle will serve well as a signal.) Let the children practice counting and timing until you feel they are skilled enough to take a measurement. Then have them count the turns of the anemometer for 30 seconds, and record on a piece of paper the number of times the red cup went around.

You may want to have the children record the wind direction as well as the wind speed. The wind vane points in the direction toward which the wind is blowing. We name the wind by the direction from which the wind is coming. In the illustration on the next page the wind vane shows that a west wind is blowing.



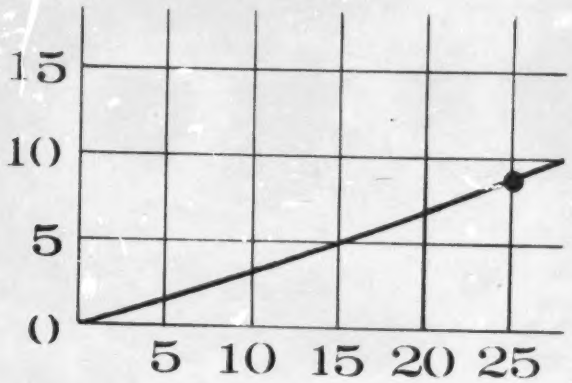
When you return to the classroom, have the children turn to Worksheet 61. This worksheet has a graph on it which enables the student to convert the number of turns he counted into a miles-per-hour wind speed reading. (If the children should ask what miles-per-hour means, say that it is the distance the air moves in an hour.)

Use the overhead projector to show the transparency of the wind-speed graph. If you have no projector, copy the graph on the chalkboard.



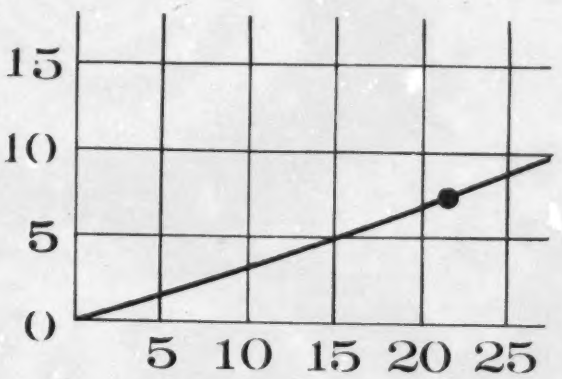
Ask how many times the anemometer turned in 30 seconds. (You might want to have the class work with a number to the nearest 5 until they have some practice reading the graph.) Trace with your finger along the "over" axis until you come to the proper number of turns. Then trace up along that grid line until your finger meets the slope line. From here you can use a finger of the other hand to trace left to the "up" axis and read the wind speed in miles per hour. Frequently this falls between two grid lines. Show how to estimate the reading by imagining 1-mile unit marks between the 5-mile unit marks that appear on the graph. (Don't try for great accuracy in this interpolation.)

The example shown below is 25 turns, which reads between 5 and 10 miles per hour. You might estimate this at 8 miles per hour.



After you have demonstrated the use of the graph, have a few children repeat the demonstration. Practice finding the wind speed for different counts of anemometer turns.

After a few readings, it should be easy for the children to begin somewhere between the grid lines for their count of the number of turns. Thus, a count of 22 turns would be shown to mean that the wind is blowing at a rate of about 7 miles per hour.



Worksheet 62 provides further individual practice in using the graph.

Worksheet 62
Unit 20 Name _____

Use the graph on Worksheet 61 to do these problems.

Number of turns	Wind speed
35	15
15	5
60	35
50	27
37	16-17
23	8
47-48	25
57-58	33

Activity C

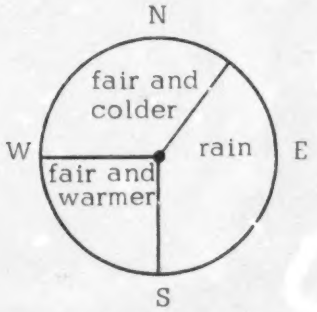
Have various groups take wind speed readings on the four sides of the school building and compare them. They should notice that some places are more protected from the wind than others. This would be a good time to introduce the terms windward (toward the wind) and leeward (away from the wind).

Now the children can add a daily wind speed reading on their Daily Weather Records. The wind speed should always be measured at the same time each day and in the most open place accessible to the children.

After the children have made the wind speed reading for each day, have them return the anemometer to the weather station with the thermometers.

Activity D (Optional)

There sometimes is a pattern to the relation between wind direction and the weather conditions that follow within 24 hours. This pattern varies in different parts of the country. Some children might want to keep a record of wind direction and weather conditions for several weeks. They could illustrate the pattern in a chart like the one below.



Lesson 18: INTRODUCING HUMIDITY

In this lesson the children are introduced to the idea of moisture in the air. Several activities show that there can be different amounts of moisture in the air. The children also learn that humidity can be measured.

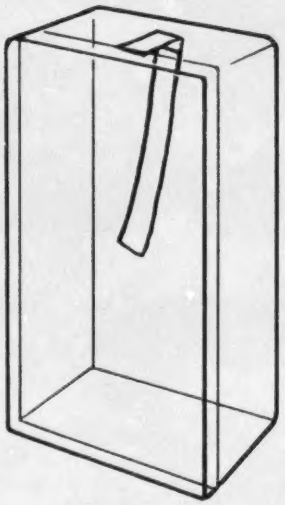
The activities in this lesson are based on the fact that cobalt chloride is a water indicator. Dry cobalt chloride paper will be blue. The paper turns bright pink when water touches it. Various shades between the blue and the bright pink indicate various proportions of moisture in the air.

MATERIALS

- 2 plastic shoe boxes
- 1 sheet of blue cobalt chloride blotting paper
- masking tape
- container of water
- sponge
- soufflé cups containing small amounts of mineral oil, white corn syrup, vegetable oil, milk, pop, water, and any other available liquids. Use a felt tip pen to label each soufflé cup.
- 200 strips of cobalt chloride test paper, $\frac{1}{4}$" x 2"
- toothpicks
- 3 twelve-ounce plastic tumblers, or jars or water glasses
- 3 petri dishes, or jar covers, or pieces of very heavy cardboard
- 1 medicine dropper
- Worksheet 63

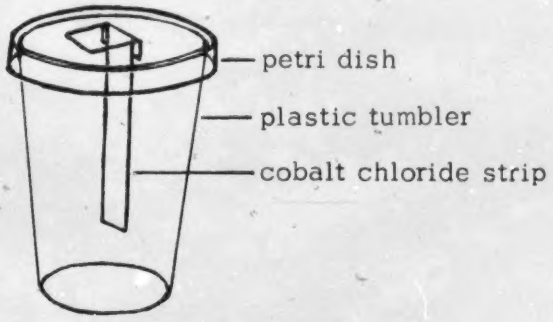
PREPARATION

For the demonstration in Activity A, you will need to tape a ½" x 6" strip of blue cobalt chloride blotting paper inside each of the two plastic shoe boxes. Tape it in the middle of one end, as shown, so that it hangs down freely.



For Activity B you need soufflé cups containing the liquids specified on the previous page. Label each cup with the name of the liquid it contains.

For Activity C, assemble three plastic tumblers, three petri dishes, and three ½" x 3" strips of cobalt chloride blotting paper in the following manner.



PROCEDURE

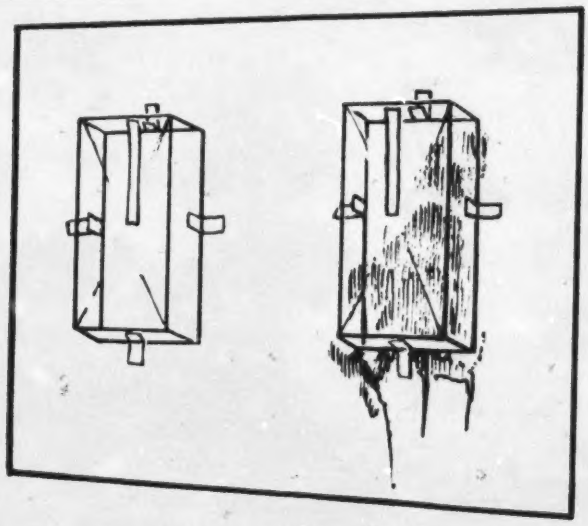
Activity A

Gather the children in front of the chalkboard, so that everyone has a clear view of it. Show the class the two plastic shoeboxes with the blue cobalt chloride strips hanging down. Have someone describe the shoebox set-ups. (They should see that both set-ups are the same.) Explain to the class that the blue paper strips have been treated with a chemical called cobalt chloride. Tell the class that you are going to use the sponge to make a wet area on the chalkboard, and then you are going to tape one shoebox set-up over the wet area and one shoebox over a dry area.

Then say:

> A MYSTERIOUS THING WILL HAPPEN TO THE STRIPS OF BLUE COBALT CHLORIDE PAPER, SO WATCH THEM VERY CLOSELY. WHEN YOU SEE SOMETHING HAPPENING, DON'T SAY ANYTHING, BUT RAISE YOUR HAND.

With masking tape, fasten one shoebox over a dry area of the chalkboard. Then wet an area about 6" x 12" and tape the other shoebox over this area. Make sure the blue cobalt chloride strip does not touch the wet board. (When you wet the board, be sure to leave it dry at the places where you will be applying the tape.)



In about 3 minutes, a noticeable change will occur in the blue strip over the wet area. In 6 or 7 minutes, the entire strip will have turned pink. Ask the children what happened.

> CAN YOU THINK OF ANY REASON WHY THE BLUE COBALT CHLORIDE STRIP TURNED PINK ON ONE BOX AND DID NOT IN THE OTHER BOX? (Let the children speculate. You may want to write their hypotheses on the board.)

If the children have difficulty making suggestions, ask them what became of the water on the chalkboard. Then ask where the water goes when a kettle boils. (Into the air.)

When you hang wet laundry on the line to dry, where does the water in it go? (Into the air.) These examples should convince the children that there is water in the air even if they cannot see it. Wet another small area on the chalkboard. Use a book to fan the wet area. Ask the children what is happening to the water on the board. Lead them to the conclusion that the water is probably going into the air.

Guide the children to the following hypothesis:

IF MOISTURE GOES INTO THE AIR, PERHAPS THE MOISTURE TURNED THE BLUE STRIP PINK.

Then ask:

HOW COULD WE FIND OUT IF MOISTURE TURNS THE BLUE COBALT CHLORIDE PAPER PINK? (By wetting another piece of the blue cobalt chloride paper.)

Use a toothpick to place a few drops of water on a strip of blue cobalt chloride paper, and have the children watch as the paper turns pink.

DOES WATER TURN THE BLUE COBALT CHLORIDE PAPER PINK? (Yes.)

IN THE NEXT ACTIVITY, WE ARE GOING TO TEST SOME OTHER LIQUIDS ON STRIPS OF BLUE COBALT CHLORIDE PAPER.

Activity B

Assemble the soufflé cups of various liquids on a table. (Be sure each cup is labeled with the name of the liquid.) Tell the class that you have some smaller strips of blue cobalt chloride paper that they will use to test these liquids to see if they also turn the blue paper pink. Demonstrate for the class how to use the strips. Use a clean toothpick each time. Dip the end into a liquid and then touch the wet end to a blue strip. The children should watch closely to see if the paper turns pink.

Divide the class into six groups and give each group a supply of toothpicks and blue cobalt chloride test papers.

Worksheet 63
Unit 20

Name _____

Liquid	Color of test paper
mineral oil	blue
white corn syrup	pink
vegetable oil	blue
milk	pink
pop	pink
water	pink

Instruct the children to use a different toothpick for each liquid. Have each group start with one soufflé cup of liquid. The name of each liquid should be written in the left-hand column of Worksheet 63. After testing a liquid, a member of the group should take it back to the table and exchange it for a new one. After each test, the children should record their results in the right-hand column of Worksheet 63.

After all the groups are finished, have a child from each group report which liquids turned the blue cobalt chloride paper pink, and which ones did not. (The mineral oil and vegetable oil should have left the paper blue, and all the others should have turned it pink.) Record the data on a class chart on the chalkboard.

Bring out in a discussion the idea that those liquids which do not turn the paper pink do not contain water. Those that do change the paper to pink probably contain water. (Other liquids might also cause the change, but tests by scientists show that only water does.)

Let the children take the remaining strips of blue cobalt chloride test papers home. They might want to test some other liquids not used in class and report back to the class the next day. Also send a copy of the Letter to the Parents home with each child.

UNIVERSITY OF Minnesota

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER
720 WASHINGTON AVENUE S.E. · MINNEAPOLIS, MINNESOTA 55414

Office of the Director

Dear Parent,

Your child has been using strips of blotting paper treated with cobalt chloride, like those he is bringing home, to test the water content of various liquids. Encourage him to tell you about the tests he has made at school and, if possible, provide him with small amounts of other liquids to test for water content. Have your child show you what happens to the color of the strips when he breathes on them, when they are placed in a steamy room, etc.

In future MINNEMAST lessons, your child will be studying the water content of the air by means of a simple hygrometer that consists of two thermometers. You will enjoy hearing him tell about this instrument and other weather-measuring instruments in the weeks to come. Your interest in your child's studies will make them more enjoyable for him, and your active participation is greatly appreciated.

Cordially,

Teacher

Activity C

Have the class gather around the demonstration table in such a way that everyone has a clear view of the table. Review Activity A, discussing the fact that when there was moisture in the air the blue cobalt chloride paper turned pink. Then ask:

IS IT POSSIBLE TO HAVE DIFFERENT AMOUNTS OF WATER IN THE AIR? (The children might talk about humid or "sticky" days in the summer, fog, how the air feels before a rain, the use of humidifiers in the winter, etc.)

Show the class the three covered tumblers with the blue cobalt chloride strips hanging in them.

WHAT HAPPENS TO THE BLUE COBALT CHLORIDE PAPER WHEN THERE IS A LOT OF WATER IN THE AIR? (It turns pink, as in the shoebox taped to the chalkboard.)

HOW COULD WE COMPARE THE EFFECT OF DIFFERENT AMOUNTS OF MOISTURE ON COBALT CHLORIDE PAPER? (Let the children speculate. Lead them to the idea that we could put a lot of water in one glass, a little water in another glass and no water in the third glass.)

Remove the covers and add water to the glasses in these amounts:

- first glass: about $\frac{1}{2}$" depth of water. Be sure it doesn't touch the hanging strip of blue paper.
- second glass: 4 drops of water. Use the medicine dropper to measure.
- third glass: no water.

glass I

Put the lids back on the glasses and tell the class to watch the strips of blue cobalt chloride paper very closely. As soon as the children see one of the strips turning pink, they are to raise their hands.

The first strip will start turning pink after approximately 2 minutes; it will turn completely pink in approximately 8 minutes. The second strip will become pale blue after approximately 10 minutes; it will turn almost completely pink after 30 or 40 minutes. The third strip will remain blue.

Keep a record on the chalkboard of the amount of time it takes for each strip to turn pink.

Strip	Time to turn Pink
1 (½" water)	8 minutes
2 (4 drops)	approximately 60 minutes
3 (no water)	no change

After 15 minutes, have the children compare the color of the second strip with the color of the third strip. (At this point, they should be able to see the second strip starting to get lighter and a little bit pink. The third strip will be completely blue. The first strip will be completely pink.) Ask the children the following question:

WHY DO YOU THINK THE FIRST STRIP IS PINKER THAN THE SECOND STRIP? (Probably because there is more water in the first container; therefore, more water can go into the air. The more water there is in the air, the pinker the paper will be.)

WHY DO YOU THINK THE THIRD STRIP HASN'T TURNED PINK? (There isn't enough water in the air to turn it pink.)

Assign someone to watch the second strip until it turns completely pink. Then you can complete the chalkboard chart.

The class may be interested in seeing what happens to a wet, pink cobalt chloride strip as it dries out. (If it was not soaked in water for a long time, it will turn blue. The color change of the cobalt chloride strips is reversible.)

Tell the children that tomorrow they are going to learn about another instrument that is used in a weather station. This one will help them measure the amount of moisture in the air.

Lesson 19: MEASURING HUMIDITY

In preceding lessons the children measured temperature and wind speed. Both of these properties of air are easily observed and the child has an intuitive awareness of them. In this lesson, the measuring process is applied to humidity.

An instrument used to measure relative humidity is called a hygrometer. The one we use in this lesson is a wet and dry-bulb hygrometer. It works on the principle that evaporation is cooling, and that evaporation is more rapid when there is less moisture in the air. We can determine humidity by comparing the temperature reading of a dry-bulb thermometer with that of a wet-bulb temperature. The difference will be more or less, according to how fast the moisture on the wet-bulb thermometer is evaporating. A humidity chart provides an index for making humidity readings. The figures are rough approximations. (Do not try to explain this to the children. Just tell them that the temperature difference is greater on a dry day.)

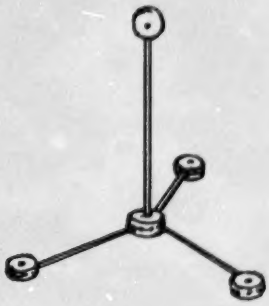
MATERIALS

- red crayons, 1 per child
- hygrometer (directions for assembling are given below)
- transparency of humidity graph (printed original included on page 121)
- Worksheets 64 through 71

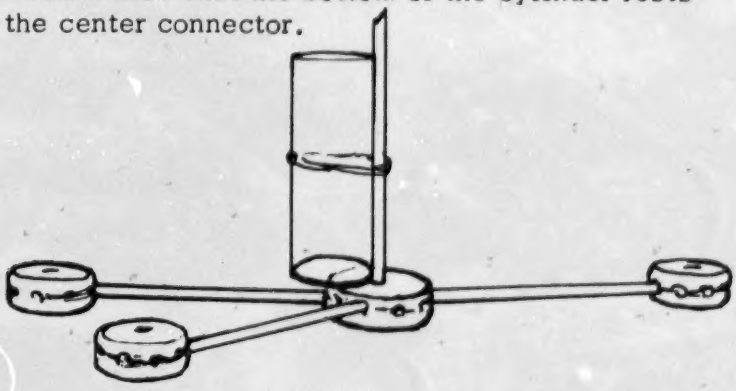
PREPARATION

Assemble the hygrometer before you conduct this lesson. The materials for constructing it are in the kit. They include: Tinkertoy rods, 3 red (about 5"), 3 yellow (about 2"), 1 purple (about 11"); 6 round Tinkertoy connectors; 2 paper clips (#1); 2 thermometers; 1 plastic cylinder ($1\frac{1}{4}$" x 4"); 1 wick ($\frac{3}{8}$" x 5"). You will also need a rubber band.

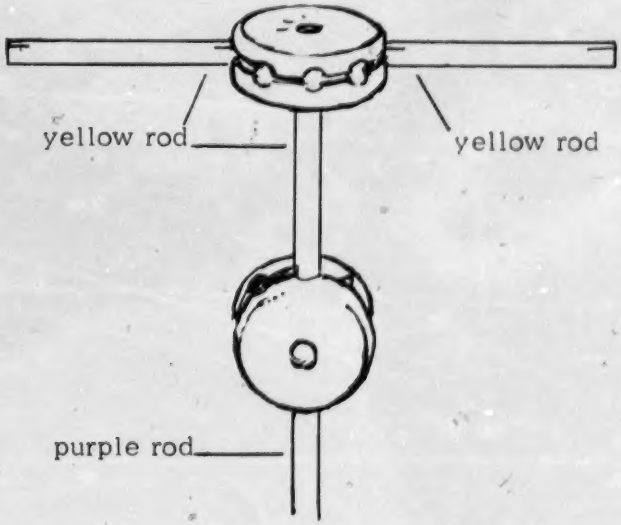
1. Connect 3 red rods to one round connector to form a base. Attach a round connector to the end of each red rod. Place the purple rod upright in the center connector and attach a round connector to it.



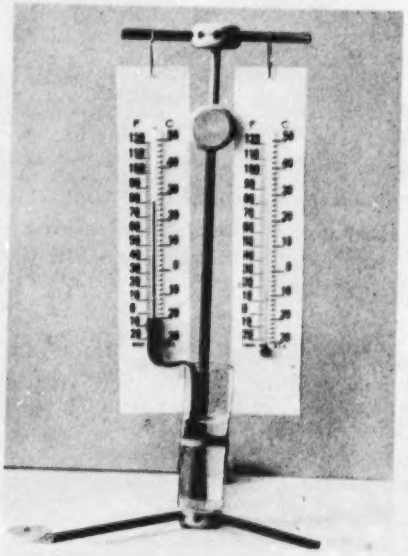
2. Attach the plastic cylinder to the purple rod with a rubber band. Make sure that the bottom of the cylinder rests firmly on the center connector.



3. Insert 3 yellow rods in a round connector, one on each side, and one in the center hole. Put this assembly in the connector on top of the purple rod.



4. Slide one end of the wick over the bulb at the bottom of one thermometer. (Be careful not to displace the column when you do this step.) Use the end of a paper clip or the point of a compass to poke a hole in the top of each thermometer backing. Hook a paper clip through this hole and slide the paper clips over the yellow rods. Secure them with tape. The wick dangling from the one thermometer should hang inside the plastic cylinder. Fill the cylinder at least half full of water. The wick should be hanging in the water.



Activity A

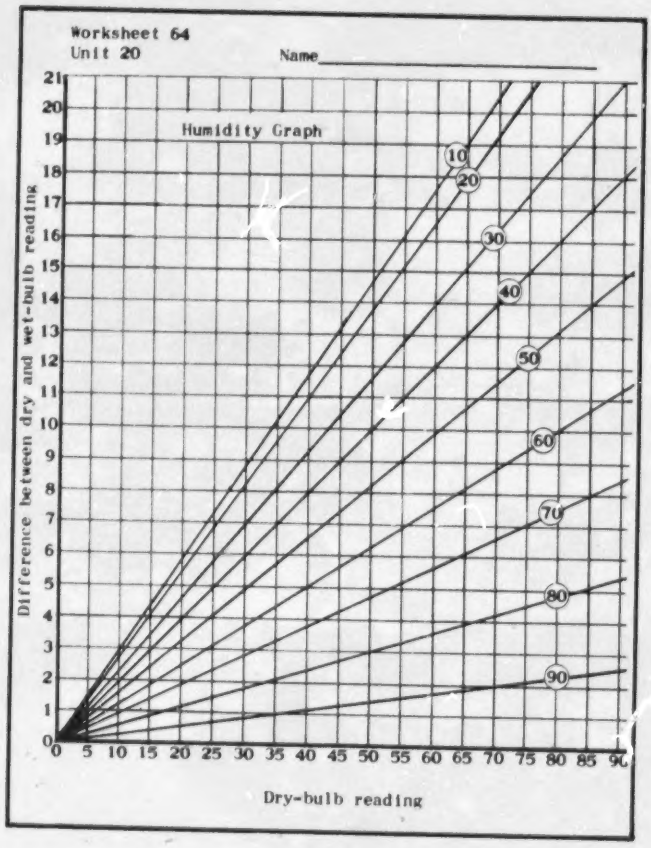
Explain that any instrument that the weatherman uses to measure the humidity, or moisture, of the air, is called a hygrometer (hy-GROM-uh-ter). Show the class the hygrometer that you made. Point out that it has two thermometers.

ARE THE THERMOMETERS DIFFERENT IN ANY WAY? (One has a wick—or shoelace—on the bulb.)

Show the children that the wick is wet.

WE CALL THIS THE WET-BULB THERMOMETER. THE OTHER IS THE DRY-BULB THERMOMETER.

DO THEY BOTH SHOW THE SAME TEMPERATURE? (No, one shows a lower temperature.)



Worksheet 65
Unit 20

Name_____

Measuring Humidity

	Dry bulb	Wet bulb	Difference	Humidity reading
Sample	10°	7°	dry 10° − wet 7° = 3°	10
Your reading	o	o	dry o − wet o = o	
Practice ①	65°	57°	dry 65° − wet 57° = 8°	60
②	47°	36°	dry 47° − wet 36° = 11°	30
③	70°	67°	dry 70° − wet 67° = 3°	85
④	72°	54°	dry 72° − wet 54° = 18°	25

Have the children turn to Worksheet 64 which is a graph for converting hygrometer data to humidity readings and to Worksheet 65, a recording sheet. The first row on this worksheet is a sample reading. Go through the following steps with the children. Have them fill in the second row on Worksheet 65.

1. Read the temperature shown on the dry-bulb thermometer, and record it in the box labeled "Dry-bulb" on Worksheet 65.
2. Fan the wet-bulb thermometer for about 15 seconds. (Use a book or a piece of stiff cardboard.) Do not have the children blow on the thermometer because the moisture in their breath will affect the reading.
3. Observe the temperature of the wet-bulb thermometer and record it in the box labeled "Wet bulb" on Worksheet 65.

4. In the box labeled "Difference," subtract the wet-bulb reading from the dry-bulb reading and record the difference.

5. Use the conversion graph (Worksheet 64) to determine the humidity reading. Project the transparency of the graph. It is used in the following way:

   a. Find the dry-bulb reading on the "over" (horizontal) axis. Hold your right finger there.

   b. Run your left finger up the "up" (vertical) axis to the numeral which shows the difference between the dry-bulb and wet-bulb readings.

   c. Run your right finger up and your left finger over until they meet. Mark the intersection with a dot.

   d. Decide which of the slope lines on the graph is closest to the dot. The numeral on that line tells the number for the amount of moisture in the air. If the dot is between two slope lines, the humidity reading is either the nearest number or, if halfway between, it is 15, 25, 35, etc.

   Example:

   Dry bulb -- 75° F.
   Wet bulb -- 65° F.
   Difference -- 10° F.
   Humidity reading -- about 60

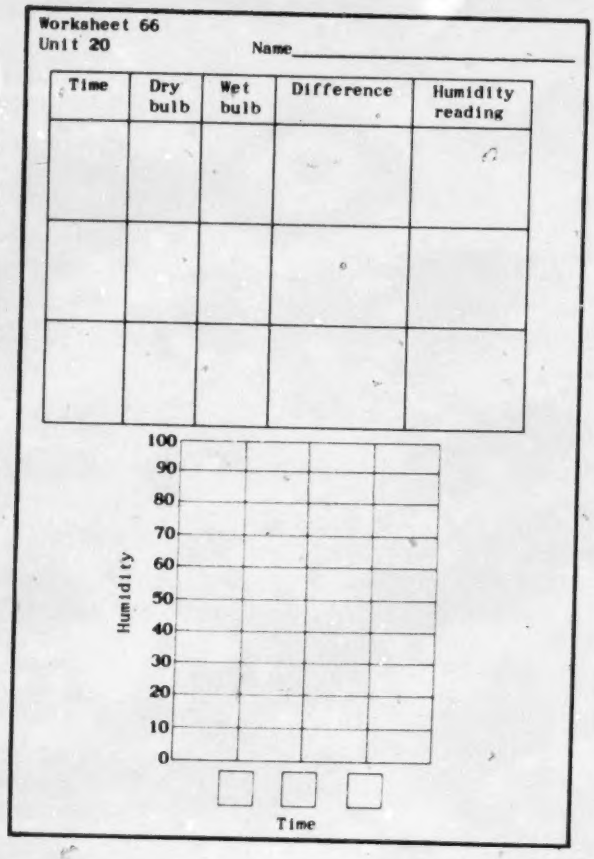
6. Record the humidity reading on Worksheet 65.

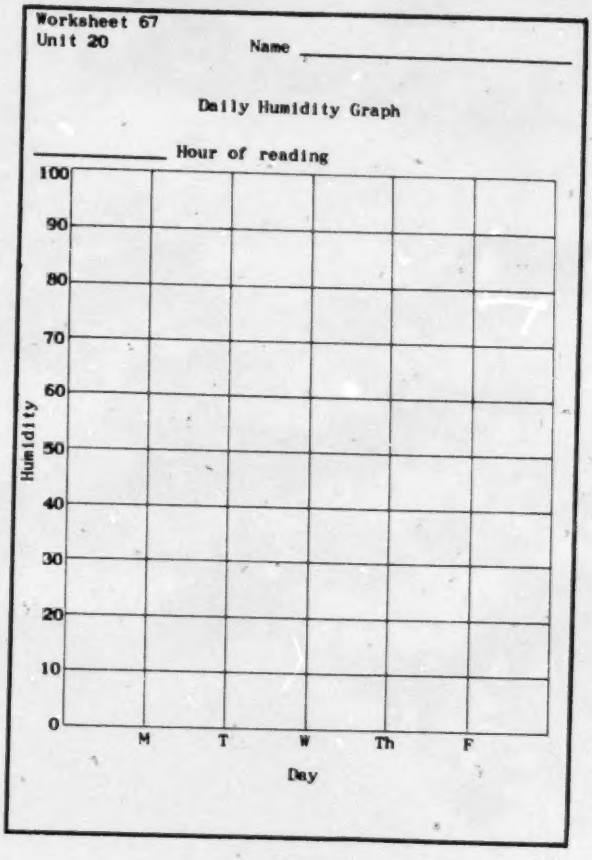
The rest of Worksheet 65 provides more practice in making humidity readings. If the children have difficulty computing the differences between the dry-bulb and wet-bulb readings, let them use their addition slide rules to find the differences.

Activity B

For one day, the children should make three outdoor humidity readings and record them in the table on Worksheet 66. Since the children have to use Worksheet 64 to make their humidity readings, have them tear out Worksheet 65. This way, Worksheet 64 faces Worksheet 66.

Use your own judgment to decide when the readings should be



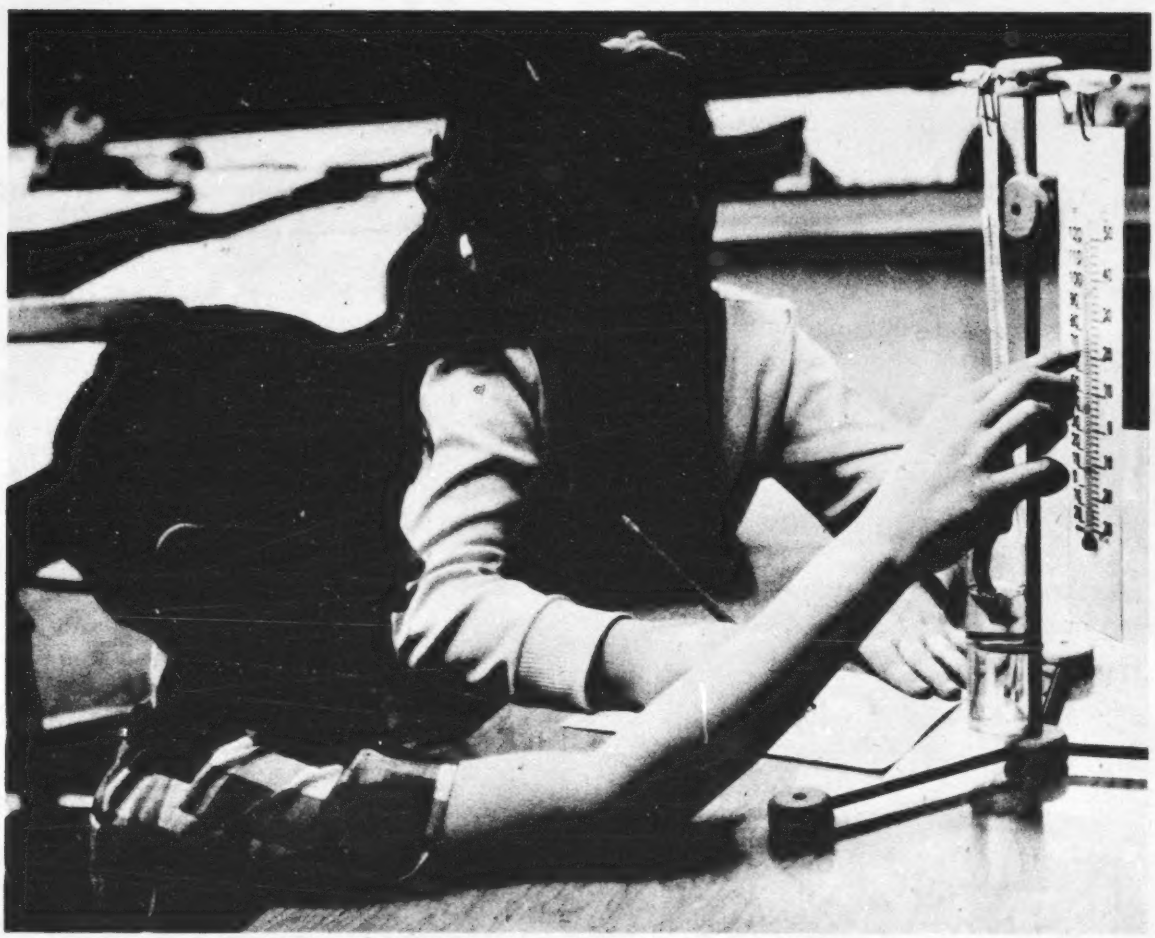


taken. You may want to have it done at regular intervals, e.g., at 10 a.m., noon and 2 p.m. However, if the weather begins to change rapidly, you may want the children to make a reading then, regardless of when the previous reading was taken. After the three readings have been made, help the children graph their data. Instruct them to fill in the time at which the readings were made in the empty boxes along the horizontal axis of the grid. Show the children how to find the time reading along the "over" axis at the bottom of the grid and the humidity reading along the "up" axis at the left of the grid. They are to trace up from one and over from the other, and at the point where the two lines intersect, make a dot or an X with their crayon. At the end of the day, they use a ruler to draw line segments between each dot and the one to the right of it. This curve shows the rise and fall of humidity throughout the day.

After the children are able to use the hygrometer with a fair degree of accuracy they can add a daily humidity reading to their Daily Weather Records (Worksheets 56, 57 and 58). Choose one hour at which they will make each daily reading. When the daily observations are made, the hygrometer should be used outdoors in a shaded area because the sun will affect the readings.

Worksheet 67 provides space for graphing daily humidity readings. The children can get the data for filling in this grid from their Daily Weather Records, after they have made humidity readings for five days. The line segments connecting adjacent readings will form a curve that shows daily fluctuations. After Worksheets 66 and 67 have been completed, discuss the graphs with the children and bring out the following generalizations:

1. The humidity may vary throughout the day;
2. The humidity may vary from day to day.



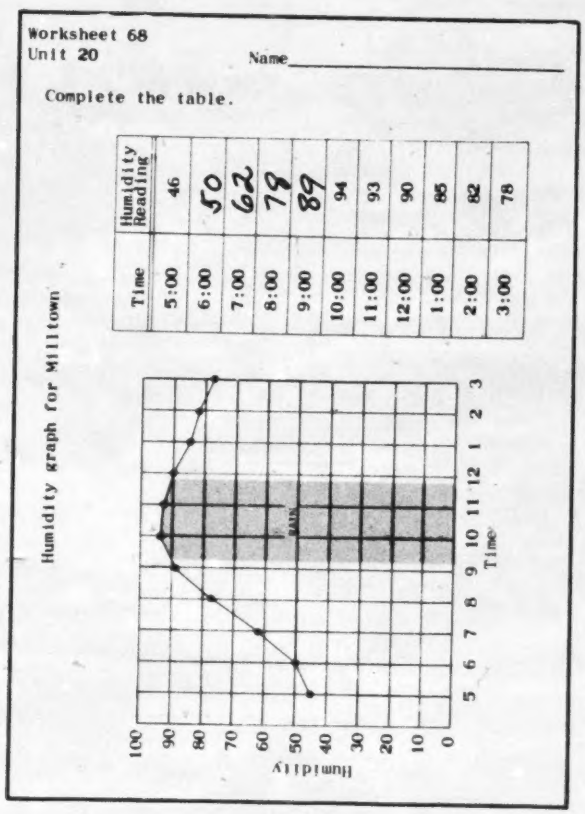
Activity C

Children should become aware that the humidity also varies in different places. Send a small group of students outdoors, another group to the school basement, and another to a hallway in the building. Have the groups measure the humidity in each location and then compare their readings.

Activity D

Worksheets 68, 69 and 70 are completed graphs of humidity readings associated with a rainstorm. The tables accompanying the graphs are partially completed. Have the children complete the tables and then tear out the worksheets, line them up and study them. On Worksheet 71, they answer questions about the information on Worksheets 68, 69 and 70. The purpose here is to have the children see that these graphs show an increase in the humidity reading before the rainstorm and a decrease after the storm.

You will notice that the graphs indicate that the humidity continues to increase briefly after the rain begins.


Worksheet 68
Unit 20 Name_____

Complete the table.

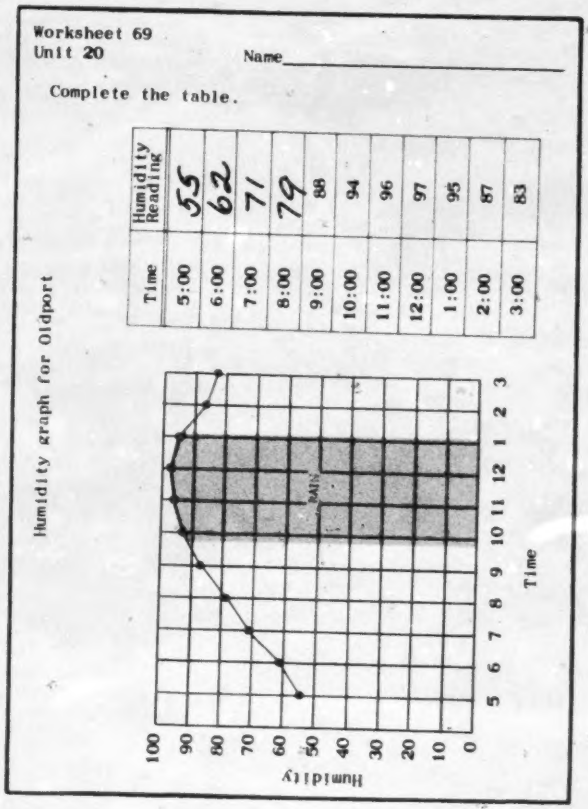
Time	Humidity Reading
5:00	46
6:00	50
7:00	62
8:00	78
9:00	89
10:00	94
11:00	93
12:00	90
1:00	85
2:00	82
3:00	78

Humidity graph for Milltown

RAIN

Time: 5 6 7 8 9 10 11 12 1 2 3

Humidity: 100 90 80 70 60 50 40 30 20 10 0


Worksheet 69
Unit 20 Name_____

Complete the table.

Time	Humidity Reading
5:00	55
6:00	62
7:00	71
8:00	79
9:00	88
10:00	94
11:00	96
12:00	97
1:00	95
2:00	87
3:00	83

Humidity graph for Oldport

RAIN

Time: 5 6 7 8 9 10 11 12 1 2 3

Humidity: 100 90 80 70 60 50 40 30 20 10 0

Worksheet 70
Unit 20 Name_____

Complete the table.

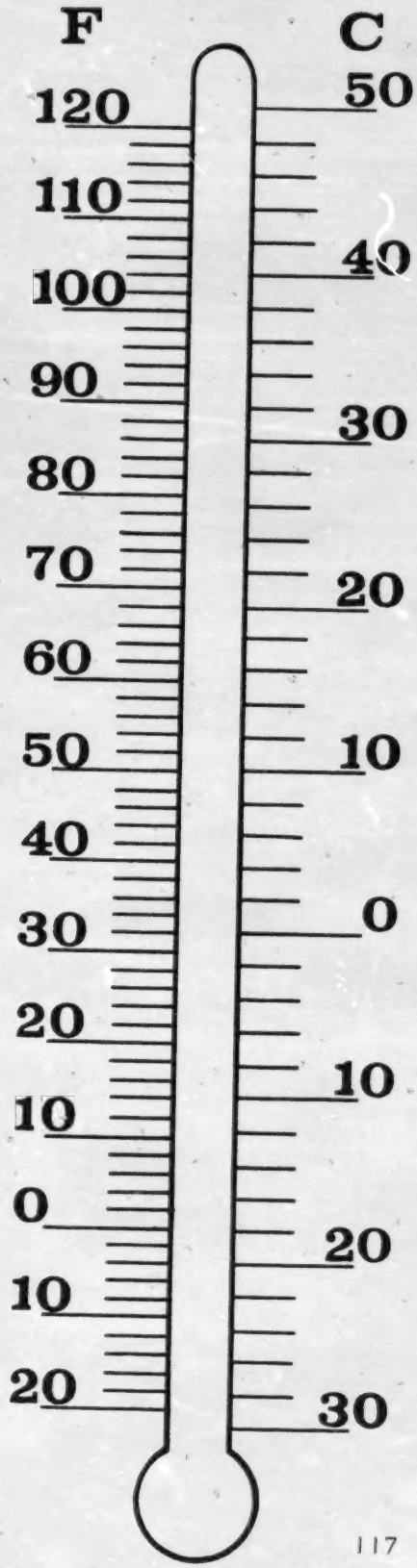
Time	Humidity Reading
5:00	70
6:00	71
7:00	72
8:00	74
9:00	78
10:00	86
11:00	95
12:00	93
1:00	85
2:00	69
3:00	47

Humidity graph for Dinkytown

Worksheet 71
Unit 20 Name _____

Study Worksheets 68, 69 and 70.
Draw closed curves around the correct answers on this page.

1. In Milltown just before the rain the humidity went (UP) DOWN
2. In Oldport just before the rain the humidity went (UP) DOWN
3. In Dinkytown just before the rain the humidity went (UP) DOWN
4. In all of the towns before the rain the humidity went (UP) DOWN
5. In Milltown after the rain the humidity went UP (DOWN)
6. In Oldport after the rain the humidity went UP (DOWN)
7. In Dinkytown after the rain the humidity went UP (DOWN)
8. In all of the towns the humidity went up before the rain. After the rain it went UP (DOWN)


F
C
120
50
110
40
100
90
30
80
70
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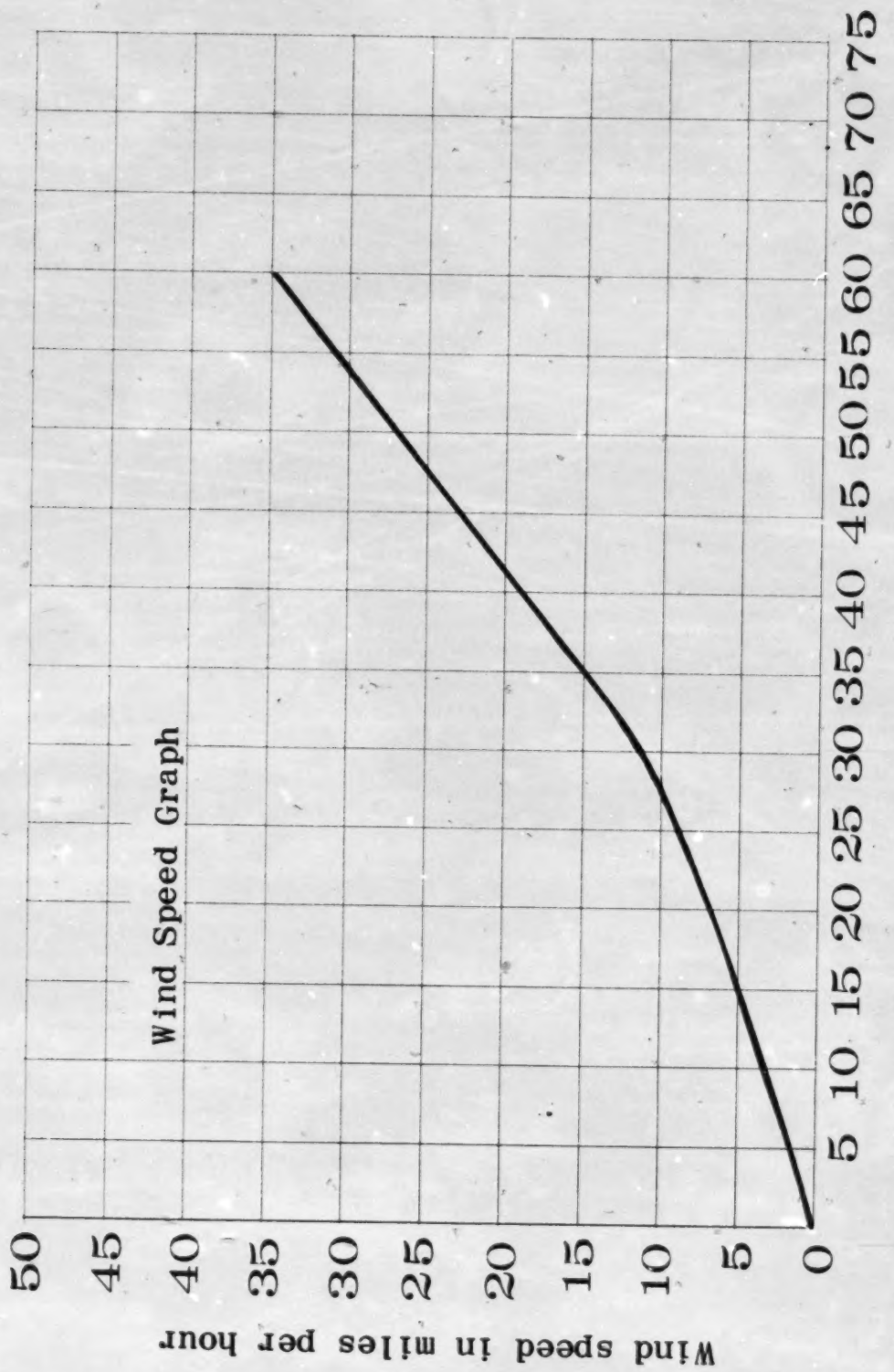
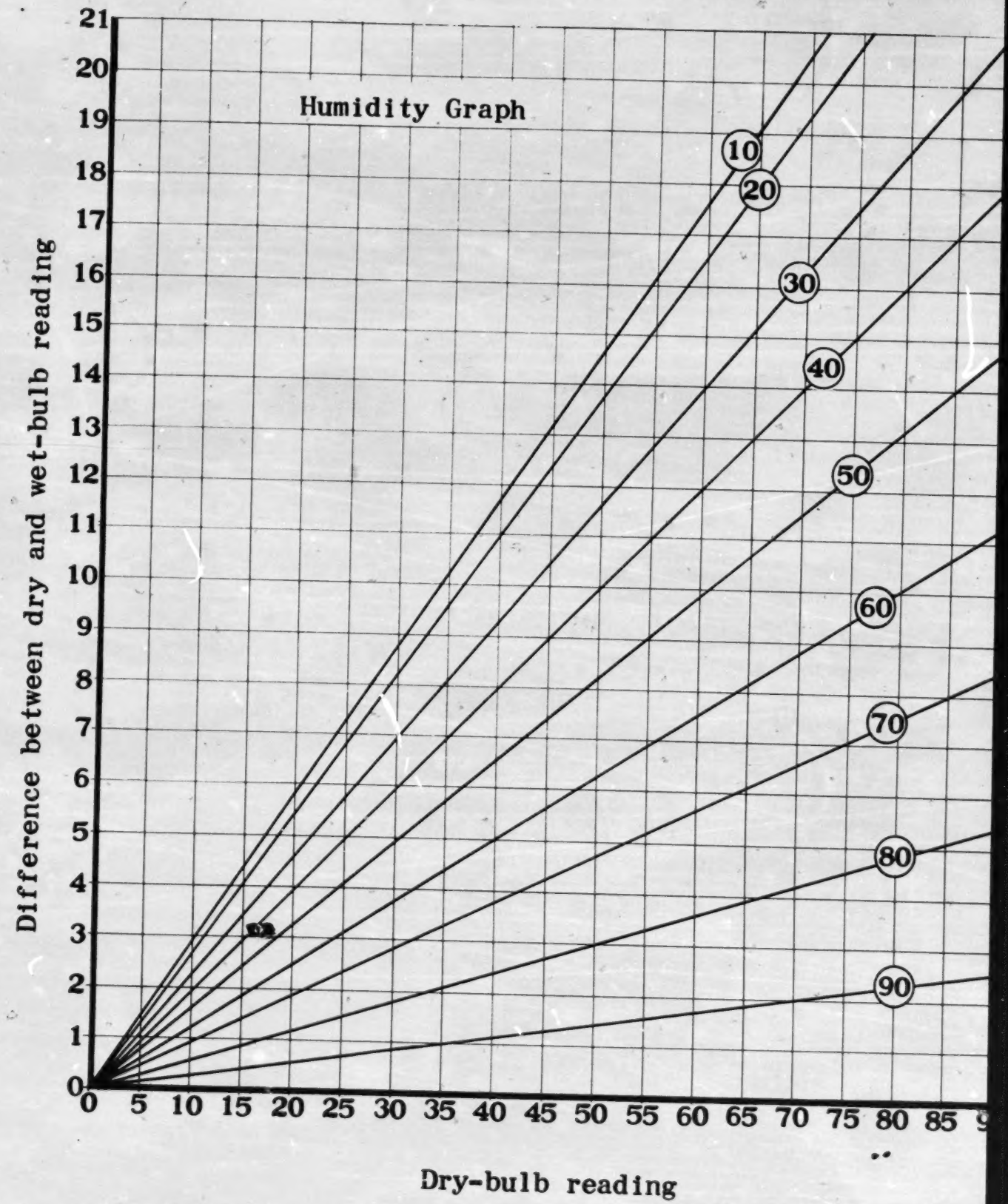
117


Wind Speed Graph
Wind speed in miles per hour
0
5
10
15
20
25
30
35
40
45
50
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
Number of turns in 30 seconds


Humidity Graph
Difference between dry and wet-bulb reading
Dry-bulb reading
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
10
20
30
40
50
60
70
80
90